C

“The acoustic impedance application”

Since nowadays apart from sound pressure, particle velocity can be determined instantly by use of the Microflown, acoustic quantities such as sound intensity, acoustic impedance and sound energy density can be measured instantaneous and at one position in space.

Sound intensity is associated with the product of sound pressure and particle velocity and quantifies the amount of sound. The specific acoustic impedance is related with the ratio of both, and is a useful quantity to determine for example the reflection or absorption of sound by matter. Sound energy density quantifies how much energy is “stored” in an acoustic wave, sound intensity quantifies how much sound energy is transported and specific acoustic impedance quantifies the possibility for sound energy to be transported.

The specific acoustic impedance at a point in a sound field is defined as the ratio of the complex amplitude of an individual frequency component of sound pressure at that point, to the complex amplitude of

the associated component of particle velocity. It is a complex number, giving the magnitude and phase of the ratio of the individual frequency components. It has an important bearing on energy flow in a sound field because the phase relationship between the pressure (a force-like quantity) and the fluid velocity clearly indicates the “effectiveness of their co-operation”, in the same way that the power factor does for voltage and current in an electrical circuit.

$$Z_s = \frac{p}{u} [Ns / m^3]$$

Here the acoustic impedance is used to determine the acoustic properties of damping material such as foam. Although a number of measurement techniques can be used to quantify the acoustic impedance of materials or structures, most often the determination of the properties takes place in a tube. This is because in a tube acoustic phenomena become one-dimensional: (in a certain bandwidth) sound waves can only propagate in one direction. This makes the experimental set-up relatively simple.

Usually an acoustic sample is put at the end of a tube and a loudspeaker at the beginning. The loudspeaker generates sound and this results in a forward travelling sound wave. A part of the sound is reflected, causing a backward travelling sound wave. The reflection coefficient is determined by finding out how much sound is travelling in the forward and backward direction. Several methods have been developed for this.

The first part of this chapter is about two methods to determine the acoustic properties of a horn loudspeaker. A traditional method (that is not used very often anymore) is compared to a novel method that is based on Microflown technology. The traditional method is based on a microphone that is moved along the axial axis in the tube to determine standing wave maxima and minima. This method is rather time-consuming.

The other method is based on a microphone and Microflown that measures the impedance of the horn directly. This proved to be a fast and reliable method.

In the second part of this chapter various methods to determine reflection coefficient will be discussed. The two microphone method is

used most commonly nowadays. A variation of that method, the two Microflown method, was examined already a shortly after the invention of the Microflown. More possible methods were developed as result of that research project. One of those methods includes a sound intensity and sound energy measurement.

Imagine the sample in the tube is full sound reflecting, so all the sound that is put into the tube will be reflected at the end. The sound intensity (the net flow of sound energy in one direction) in the tube will be zero due to this. If on the other hand the sample is fully sound absorbing, the sound intensity will be a large figure. How large this figure is depends on the amount of noise that is generated by the loudspeaker. The loudness can be determined by the sound energy density. The ratio of the intensity and the energy is zero for a full reflecting sample (intensity is zero) and (apparently) the speed of sound for a sample that is full absorbing.

C1 MEASURING THE ACOUSTIC IMPEDANCE OF A HORN LOUDSPEAKER

Two methods to measure the acoustic input impedance of a horn are compared. The first method measures standing wave patterns in a tube that is terminated by the horn. The input impedance is calculated from the position of the first minimum in the standing wave pattern, and the ratio of maximum and minimum sound pressure level in the tube. Secondly a direct (p/**u**) method is applied. The Microflown (**u**) is used together with a pressure microphone (p), which are mounted in the throat of the horn. Results from both measurements are compared with simulated models. This was the first time that a Microflown was used for measurement purposes, only a few months after its invention in 1994. This chapter is based on the work that is presented in [8].

Introduction

Horn loudspeakers are widely used in the area where one needs good frequency independent directivity of sound, together with high sound levels. Disadvantage of this type of loudspeaker is its production of excessive distortions at high driving levels. In preliminary research concerning compensation of this distortion, it appeared that the transfer characteristics of the horn itself need to be known accurately to yield better performance of distortion reduction techniques [1].

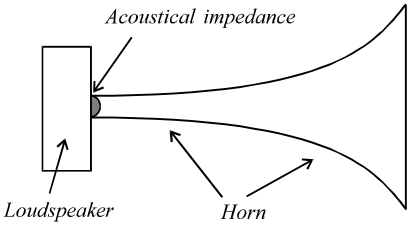

Fig. 93: A horn type loudspeaker.

The acoustic input impedance, determined by the geometrical properties of the horn, is considered in the research presented here. Measurement of this acoustic impedance is important for two reasons. In

the analysis of the behaviour of the horn–driver its acoustic load is fully determined by this input impedance. Secondly for verification whether or not models of a specific horn are accurate enough or if they need refinements.

Here two methods to measure input impedance will be compared. The first method was proposed by Fahy for measurement of loudspeaker cabinet impedances [2]. In the second method the input impedance is measured directly using a pressure microphone and the Microflown [3], both mounted in the throat of the horn.

Theory

In modelling physical systems, it is of importance to use as much as a priori knowledge as possible. That is in this case the knowledge about the geometrical pattern, i.e. the varying cross–sectional area along axial axis of the horn. Next to this there are the throat and mouth which form the boundary conditions. Basic theory, which uses the loss–free plane wave fundamental horn equation, is well-established [4], [5].

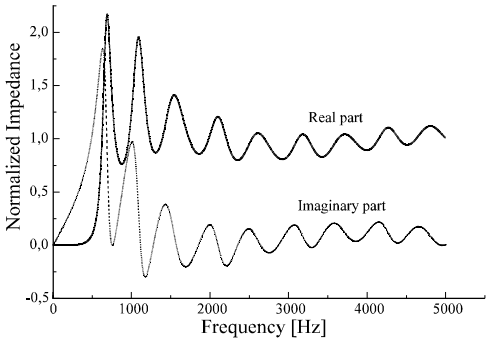

Fig. 94: The theoretical acoustic impedance of a horn.

The acoustic input impedance, the complex quotient of the sound pressure and volume velocity in the throat, is found by solving the fundamental horn equation and applying proper boundary conditions. The specifics on how to solve the acoustical impedance ($Z=R+jX$) of a horn can be found in [8], the result is shown in Fig. 94.

If the real part of the normalised impedance is one and the imaginary part is zero the maximum acoustical power is transmitted.

Measurement methods

Fahy's method

One method to measure the acoustic impedance is by measuring standing wave patterns inside a tube driven by a loudspeaker that is terminated by a certain acoustic element of which we want to determine the impedance. Fahy used this method to measure loudspeaker cabinet impedance [2], while here it will be used to measure the input impedance of a horn. The measurement set–up is depicted in Fig. 95.

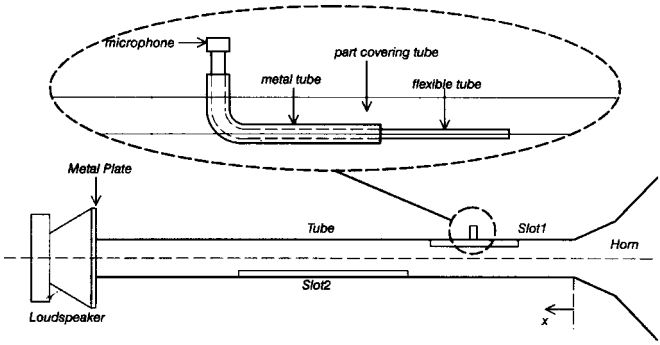

Fig. 95: Fahy's measurement set up.

A loudspeaker is attached to one end of a brass tube, while the other end is terminated by the horn. Diameter of the tube is chosen such that there is a smooth transition from tube to horn throat, while the length of the tube has to be at least one–half wavelength of the lowest frequency of interest. A probing microphone is needed for measuring the sound pressure inside the tube. It should also be moved along the axial axis to determine standing wave maxima and minima. Therefore two slots are made where the probing microphone can move along, keeping the slot as tight as possible closed by means of a covering tube. Two slots are necessary because we need a shorter covering tube to measure near the throat of the horn. For the driving loudspeaker a normal full range electrodynamical loudspeaker was chosen. It is not possible to find such a loudspeaker for the range of $300Hz–5kHz$ with a cone diameter of

$22mm$, and therefore a flange was made to connect loudspeaker to tube. The hereby-created volume in front of the loudspeaker has a Helmholtz resonance frequency of $218Hz$, which is therefore of no concern. The lowest cut-off frequency of the tube, which has a diameter $d=22mm$, is given by:

$$f_c = \frac{0.6c}{d} \tag{156}$$

which yields a maximum frequency of $9.2kHz$. This is sufficiently high for a frequency span of $300Hz–5kHz$. The length of the tube is $610mm$, $which$ yields a lower frequency of $283Hz$. For determination of the input impedance three values have to be determined. First the position of the sound pressure minimum nearest to the throat, x_{min}, and next the maximum and minimum sound pressure of the standing wave pattern in the tube, p_{max} and p_{min}, are determined. The complex acoustic impedance is then found from [2]:

$$Z = R + jX = \frac{\left(\frac{1.52c}{d^2}\right)(1-r^2)}{1+r^2-2r\cdot\cos(t)} + j\frac{\left(\frac{1.52c}{d^2}\right)2r\cdot\sin(t)}{1+r^2-2r\cdot\cos(t)} \tag{157}$$

With r the pressure reflection coefficient and t the reflection phase angle which are given by:

$$r = \frac{a-1}{a+1}; a = \frac{p_{\max}}{p_{\min}}; t = \pi\left(\frac{4x_{\min}}{c}+1\right) \tag{158}$$

Because only the ratio of maximum and minimum pressure is required, the microphone does not need calibrating. This procedure is performed for every frequency in the span of interest. Because it is necessary to seal the tube properly with each measurement, it more straightforward to perform a full frequency sweep of 401 frequency points at every measurement spot in the range of $x=0–393mm$, with a resolution of $1mm$. This is a very tedious task that yields an enormous amount of measurement data, especially if measuring as accurate as possible, which in this case means 394 separate measurements.

Direct method

Most straightforward method to measure acoustic impedance is of course direct measurement of sound pressure and particle velocity inside the throat of the horn. Sound pressure can be measured using a calibrated microphone, but a particle velocity sensor for midrange frequencies was not available until recently [3]. The Microflown was in this case calibrated by means of a time frame measurement inside a very long tube [3], see also chapter A5 "*calibration methods*". The Microflown that was used at that time is depicted in Fig. 4.6.

Both, microphone and Microflown are mounted in the throat of the horn, thereby firmly sealed to the throat by modelling clay. Influence of the Microflown on the sound waves is negligible small as its dimensions are very small. The microphone has to be placed with more care because its dimensions are such that sound pressure distribution may be disturbed. Best compromise between accurate measurement and least disturbance is found by placing the microphone end flush with the inside of the horn wall. It was hereby assumed that the pressure in the throat of the horn is equally distributed along the cross sectional area, as in the throat we have plane wave propagation. Major advantage of this method compared to the previous one, is that the measurement set–up is much simpler and it is performed by one single frequency sweep measurement where an analyser measures both complex pressure and flow signals.

Results and discussion

Measurement results of both methods are depicted in Fig. 96 and Fig. 97. Comparing them with each other, we notice directly a much noisier result with Fahy's method. This is due to the fact that this result is obtained from a lot of distinct measurements which yields "position noise", which on its turn affects the calculated impedance from it. This is why measurements are limited to the frequency span of $383–4500Hz$, outside this span measurements were not useful.

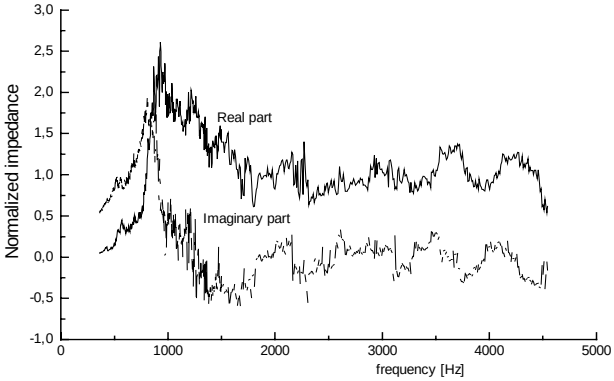

Fig. 96: The acoustic impedance of the horn input impedance measured using obtained by Fahy's method.

Looking at Eq. (8), the sensitivity for an error in x_{min} is quite high especially whenever t is near to $(\pi/2+n\pi)$ *or* $(n\pi)$, $n\in 1,2,...$ Next to this a difference in the frequencies was observed where the first maximum in the resistance and reactance occurs. This means that a higher cut–off frequency was measured with Fahy's method. Despite the differences there is, of course, resemblance between both measurements. Most maxima and minima in both resistance as well as reactance have the same quantities and both measurements show a qualitative great resemblance.

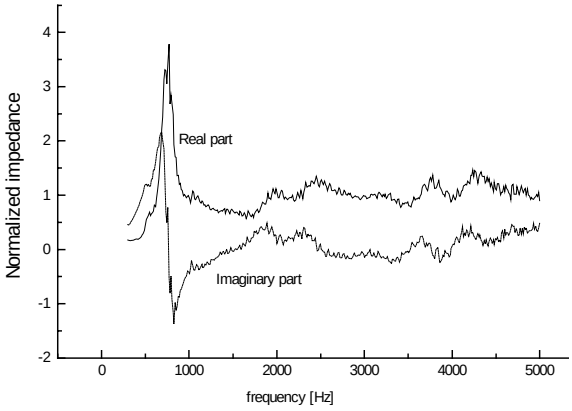

Fig. 97: The acoustic impedance of the horn input impedance measured using obtained by the direct method.

A comparison between measurements and the model is of even more interest. Although the results are noisy, it is clear that for frequencies above *2kHz* measurement and model have a fairly great resemblance. Below this frequency however, there is a great error that is clearly caused by a shift of the first maxima in frequency. Possible cause for this discrepancy is the influence of the horn input impedance on the motion of the loudspeaker cone [2]. This is the case when the acoustic reactance of the loudspeaker is equal to or smaller than the horn impedance. For the applied driving loudspeaker (*d=80mm, m=4.5g*), this yields a frequency of up to approximately *2kHz* where the influence is significant. Still, the qualitative agreement above this frequency is fairly good, especially for the resistance. A much greater resemblance, both quantitative and qualitative, is found when comparing the model with the direct measurement results, which are depicted in Fig. 97. Still, a difference is noticed between the frequencies where the first minima and maxima occur in the lower frequencies, but it is much smaller. Up to approx. *500Hz* measurement and model almost coincide (reactance in particular), while between *500Hz* and *600Hz* both the resistance and reactance become almost constant for about 100Hz.

It is difficult to state what causes these differences between model and measurements. Fact is that the deviation with the direct method is less than with Fahy's method. It is assumed that the influence of the microphone probe and slots on the tube do not disturb standing wave patterns. Next to that there are the great sensitivity towards errors in x_{min}. It is therefore concluded that Fahy's method is less suitable for measuring the input impedance of a horn. Discrepancy between the direct measurement and model are not caused by these interferences. Single assumption made in this measurement is that waves inside the throat of the horn are not disturbed by the placement of the microphone and Microflown.

Finally one could state that the model is not accurate enough. The horn used in this research has directing elements inside the mouth of the horn that may affect its impedance. These elements are not taken into account in modelling of the horn.

Conclusion

Major conclusion drawn from this research is that direct measurement of (horn input) acoustic impedance is performed faster, easier, and more accurate by using a Microflown. Using the transmission line approximation for horn modelling [8] together with this measurement a valuable tool for modelling horns and other acoustic structures is obtained. Any horn can be approximated by this method and its model parameters easily tuned to obtain resemblance between measurement and model. Also it appeared that Fahy's method is useful for measuring the input impedance of the horn, but is very sensitive to errors in x_{min}. Next to that one needs to perform a lot of tedious measurements in which surround noise and system varying conditions (temperature, humidity etc.) affect the measurement. Still, Fahy's method can be seen as a complementary method next to the direct method, especially at higher frequencies (remind that a Microflown from 1996 was used here, it has improved much since then).

Discrepancies between model and measurement are caused by elements that were not taken into account, like the directing elements in the mouth of the horn. Additional applications of the Microflown used in this research, are in linearising methods where a prediction of the particle velocity in the throat of the horn is needed to compensate nonlinear propagation of sound waves inside a horn [7]. We could use the velocity signal as a feedback signal to avoid modelling or equalisation of the horn driver transfer function.

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C2 METHODS FOR MEASURING THE REFLECTION COEFFICIENT OF SOUND ABSORBING MATERIALS

In this chapter several techniques to measure the sound absorption coefficient are described. Three of the techniques are new and make use of the Microflown. It will be shown that a combination of a microphone and a Microflown provides direct information on the acoustic impedance, the sound intensity and the sound energy density. Experimental results of these four methods are compared to each other. A well-described test sample with a quarter-wave resonator is used to be able to repeat measurements in a reliable way. This chapter is based on the work that is presented in [35], [36], [40], [41] and [42].

Introduction

A number of measurement techniques are available to quantify the sound absorbing behaviour of porous materials [42]. Obviously these techniques can be used as well for a wall with resonators. Here a brief overview of measurement techniques for sound absorbing materials is given.

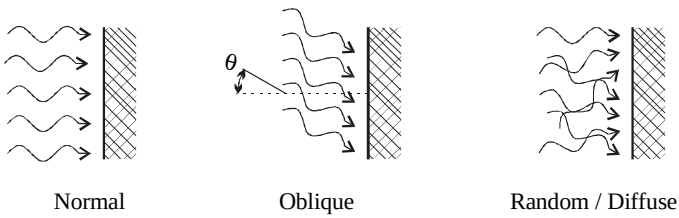

Fig. 98: Three types of incident waves.

In general one is interested in: the sound absorption coefficient α, which is the fraction of the total incident sound energy which is dissipated in the porous material, the reflection coefficient R, or the normal surface impedance Z_n. These quantities are usually measured for

normal incident waves. The incident sound field can be classified in three types: normal, incidence and random incidence, see Fig. 98.

Typically the absorption coefficient increases with an increasing angle of incidence up to a certain angle. Beyond this angle a decrease is usually observed. The explanation for this is the contribution of the so-called shear waves that propagate in the flexible porous material. As a result the absorption coefficient at normal incident is slightly less than the absorption coefficient measured at random incidence for porous materials. The normal impedance on the other hand is a complex vector that is oriented normal to the surface of the porous material and directed inward. In this case one can speak of the *normal* surface impedance of a material measured with *oblique* incident sound waves. The acoustic measurement techniques can be divided into three categories: reverberant-field methods, free-field methods and impedance-tube methods (Kundt's tube).

Reverberant field method

The so-called reverberant field method is a well-known technique to measure sound absorption coefficients for random incident waves [42]. Experiments are performed in a reverberation chamber in which a diffuse sound field is generated (see for example Bies 1996, ISO 354).

There are a number of standards available for the procedures as well as for the geometry and dimensions of the test chambers. Usually a sound pressure field is generated with a uniform energy density. This is achieved with loudspeakers that are placed in the corners of such chambers and a number of diffusers to prevent the presence of standing waves in the chamber. A relatively large sample of the sound absorbing material (several m^2) is placed in the chamber and for a given frequency band the reverberation time T_{60} is measured. T_{60} is the elapsed time at which the sound pressure level has dropped 60dB after the shutdown of the loudspeakers. The same procedure is performed without the sample and the difference is a measure for the absorption coefficient.

For highly sound absorbing material the absorption coefficient can exceed a value of one because of extra energy loss due to edge effects and diffraction. This can also be the case if the sound field is non-diffuse. Various standards state that at least 20 modes of vibration in the chamber are required in the lowest frequency band. As a result the room volume must be quite large. Nevertheless considerable differences have been

observed for measurements on the same test materials in different reverberation chambers. Although it is the only method to apply diffuse sound fields, it is concluded that the reverberant field method is less suitable for testing samples with broadband resonators.

Free field methods

Free field methods are commonly used for radiation measurements of sources of sound [42]. The free field condition indicates that waves only propagate directly from the source of sound. This condition can be realised in an anechoic chamber. In practical situations there is usually reflection from the ground. For suchlike situations outdoor measurements above a reflecting plane can be made or a semi-anechoic chamber can be used where the floor is a reflecting plane.

A number of authors have proposed methods to measure the acoustic properties of sound absorbing materials under free field conditions (see for example Tamura 1995 and 1990, Allard 1989-1 and 1989-2). In general the methods are suited for measurements with oblique incident waves. One technique is for example the pulse technique. A short signal is generated and the direct and the reflected waves are separated to calculate the reflection coefficient. It is noted that the sample has to be placed outside the near field, which can pose a lower limit on the frequency band of interest, and on the dimensions of the samples (several m^2). Another technique uses two microphones placed close to the sound absorbing surface. With this method it is possible to calculate the normal impedance at the surface for oblique incident waves. The area of the test material can be much smaller ($1m^2$). For lower frequencies however the size of the anechoic chamber may be a restricting factor because the source should be placed outside the near field.

The possibility to measure the acoustic behaviour of sound absorbing materials under oblique incident waves is a strong advantage of the free field method. It was already mentioned that for oblique incident waves the shear waves that propagate in the sound absorbing material cause a different acoustic behaviour. However for the material tested with tube-resonators the shear waves are not present. Furthermore the resonators are locally reacting. Therefore it will be shown that it is sufficient to use the impedance tube technique to measure the normal impedance of a wall with a number of resonators.

Impedance tube methods

Although a number of measurement techniques can be used to quantify the sound absorbing behaviour of porous materials, most often the determination of the properties takes place in a tube [42]. This is because in a tube the problem becomes one-dimensional: (in a certain bandwidth) sound waves can only propagate in one direction. This makes the experimental set-up relatively simple. In Fig. 99 a sketch of two techniques is shown.

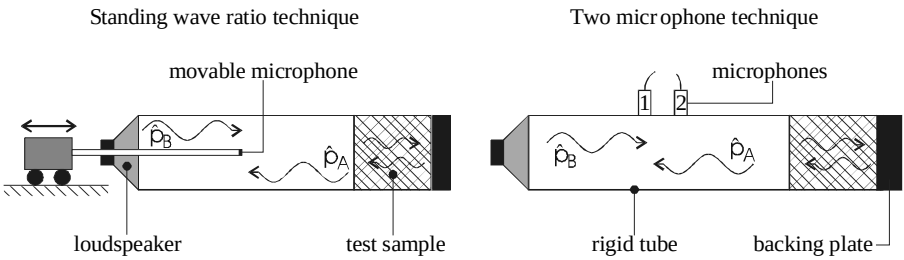

Fig. 99: Schematic representation of two measurement techniques in an impedance tube.

At the left-hand side a loudspeaker is placed and at the opposite side a sample of the test material is placed. In the tube a standing wave pattern is formed, being the result of a forward (or incident) travelling pressure wave with amplitude $\hat{p}_B$, and a backward (or reflected) wave with amplitude $\hat{p}_A$. The frequency of the sound waves is kept lower than the cut-off frequency to assure the generation of plane propagating waves in the tube.

The one microphone method

Earlier techniques made use of the measured standing wave ratio (SWR) for a specific frequency in the tube. By means of a movable microphone the ratio of the pressure maximum to the pressure minimum is determined. This ratio is then used to calculate the reflection coefficient and the acoustic impedance. An advantage of this method is that it is not necessary to calibrate the microphone. Drawbacks are the complex set-up with a movable probe and time needed to find the

maximum and minimum pressure for *each* frequency of interest. This method is described in chapter C1: "*Measuring the acoustic impedance of a horn loudspeaker*".

Two microphone methods

In 1980 Chung and Blazer [43], [44] resented a technique which is based on the transfer function of two fixed microphones which are located at two different positions in the tube wall (see right-hand side of Fig. 99) [42]. This method will be called the **2p method**. The standing wave pattern in their case is built up from a broadband stationary noise signal. With the measured transfer function incident and reflected waves are separated mathematically. This leads to the reflection coefficient of the sample for the same frequency band as the broadband signal. The impedance and absorption coefficient can be derived as well. The method is as accurate as the SWR method and considerably faster. The transfer function method has proven to be reliable and has been standardised (ISO 10534-2, 1998).

For sound absorbing materials the impedance measured with the method as described above strongly depends on the thickness of the material because the sound waves reflect at the backing plate. Therefore some authors advise the use of acoustic properties which are independent of the test configuration such as the characteristic impedance and the propagation coefficient in the material (see for example: Delany [45], Minten [46], Lauriks [47], Utsumo [48], Voronina [49], Iwase [50], Song [51]). One technique to derive these two coefficients is to measure the surface impedance of the material with two different thicknesses.

For low frequencies the impedance tube method may not give accurate results because an airtight fit of the sample is needed and at the same time the sample has to be able to vibrate freely. This may also be a problem for higher frequencies when laminated materials or materials covered with a screen (for example a perforated sheet) are used. Furthermore, for a non-zero transverse contraction ratio (Poisson's coefficient) it is unlikely that a small sample is representative for a large area. For rock and glass wool however Poisson's coefficient is approximately zero.

Analogously to the 2p method it is also possible to use particle velocity sensors to determine the impedance Z (**2u method**). This method

is presented in [40]. Again the transfer function between the two (particle velocity) microphones is dependent on the acoustic impedance *Z*, the spacing between both sensors and the distance between the sample and the sensors. Advantage of a 2u method compared to the 2p method is that it is (contradictory to pressure microphones) quite easy to make two Microflowns with similar properties.

It is also possible to use a combination of a sound pressure probe and a particle velocity sensor and measure the impedance in the tube in a direct manner. This method is in fact almost the same as the 2p and 2u method: the acoustic impedance is determined measured in the tube and then it is calculated what the acoustic impedance at the end of the tube was. We will call this the **p/u method**. (In fact this is also done by Schurer et al., see chapter CI and [38]).

Traditional sound intensity determination is based on two closely spaced sound pressure probes. Sound intensity is a vector quantity and represents the propagation of sound energy. It is the product of the pressure perturbation *p* and particle velocity perturbation ***u*** (see for instance Fahy [39]). Note that the acoustic impedance is the ratio between the pressure and the velocity and is also, strictly speaking, a vector. The measurement of sound pressure is relatively straightforward but the particle velocity perturbation is usually estimated indirectly by using two closely spaced microphones. Via the momentum equation one can show that from the pressure gradient the particle velocity is obtained. The gradient is estimated with two microphones (Euler's method). This explains the *two* microphone technique.

Measuring the sound intensity has a number of advantages above sound pressure measurements. For instance the radiated sound power of a source can be measured *in situ* instead of in a reverberation or anechoic chamber because it is possible to perform measurements with stationary background noise. A large number of new techniques on sound intensity measurements is listed in the literature and a number of standards has been developed (see for example Bies [37], Isaksson [52]).

Similar to the sound intensity determination the specific acoustic impedance can be determined with the use of two closely spaced microphones; i.e. by determination of the pressure on the two places, the specific acoustic impedance is calculated from the ratio of the sum of both signals and their difference. This method is called the **p-p method**. Employing a similar procedure with two particle velocity probes is called

the **u-u method** (see also chapter B5: *"The u-u sound intensity measuring method"*).

Another way to determine the acoustic properties of a sample is to measure the product of both different sound probes: the p·**u** method. This has a lot of implications since now the *sound intensity* is determined in stead of the acoustic impedance. Contrary to the acoustic impedance, the sound intensity is a constant value in the tube: there will be no dependence of the distance the distance between the sample and the pressure sensors.

All the methods do have drawbacks:

- The **1p method** is very time-consuming and as can be seen in [6] a lot of "positional noise" will be introduced. The same problems are to be expected with the **1u method.**
- The disadvantage of the **2p** and the **2u method** is that the transfer function between the two microphones that determines acoustic impedance *Z* is also dependent on the distance between the sample and the pressure sensors. Another note of attention is that the probes should be identical. The effects of mismatch can be overcome by exchanging the microphones and do the measurements again
- The **p/u method** has the same drawback as the 2p and the 2u method: the determination of the impedance Z at the end of the tube is also dependent on the distance between the sample and the acoustic sensors. Another drawback is that the acoustic sensors are fundamentally different, so the calibration of the sensors will be a mayor concern.
- Since the **p-p method** and the **u-u method** are not fundamentally different to the 2p or the 2u method, the same problems will be encountered.
- The transfer function of the loudspeaker will also be measured using the p-**u** method. To overcome this problem, the acoustic energy must also be determined. Like the p/u method the calibration of the sensors will be a concern. It will be shown that this method reveals only the modulus of the reflection coefficient.

A possible advantage of the methods that use only microphones or Microflowns is a method that makes use of only one sensor. The measurement is repeated with an altered sensor position using two times the same input signal. The sensor signals are stored in a computer and

synchronised in post processing in such way that the measurement seems to be performed with two sensors at the same time in stead of with one sensor and in two times.

Theory

In this paragraph theory will be treated on how acoustic properties of materials can be determined. The 1p method is discussed in chapter *C1 "Measuring the acoustic impedance of a horn loudspeaker"* and [38]).

First general theory will be treated on sound pressure and particle velocity in the tube. Then the p/u method will be presented. After this it will be shown that the p-p method is in fact mathematically similar to the p/u method. The u-u method will not be given extensively because in this one-dimensional environment it is quite similar to the p-p method, see chapter B5 *"The u-u sound intensity measuring method"*.

The 2p method is known, it will be explained and it will be shown that the method is mathematically similar to the previous methods. It will also be shown that the 2u method is similar to the 2p method. At last theory and measurements of the p·**u** method will be given. A method that theoretically can determine the transmission of an acoustic sample will conclude this chapter.

General theory

The impedance tube is used to determine the sound absorbing characteristics of materials like glass-wool or foams. In the tube a one-dimensional acoustic field is generated with a speaker. The plane waves are reflected and absorbed by the sample material at the end of the tube. The reflection coefficient R or the absorption coefficient $\alpha=1-|R|^2$ can be calculated from the acoustic impedance Z. All quantities, R, α and Z are frequency dependent.

Here only the basic theory will be explained. The viscosity and the thermal conductivity of the fluid affect the wave propagation. Due to this, the measurement results will be a few percent under estimated. More correct models are presented in [41].

In a tube, above the cut-off frequency the specific acoustic impedance changes due to the existence of standing waves perpendicular to the direction of the sound wave. The set-up will then be very difficult to use

since the acoustic impedance is not a real and constant anymore. For a tube this cut-off frequency is given by:

$$f_c = \frac{c}{1.71 \cdot d} \tag{159}$$

Where d represents the diameter of the tube, and c the speed of sound (approximately 330m/s).

The specific acoustic impedance at a distance x relates the acoustic pressure $p(x,\omega)$ to the normal component of the fluid velocity $u_n(x,\omega)$ according to:

$$Z = \frac{\hat{p}(x,\omega)}{\hat{u}_n(x,\omega)} \tag{160}$$

The pressure amplitude $\hat{p}$ is (that may be complex) at a single frequency of the pressure p:

$$\hat{p}(x,\omega) = p_0 \; e^{i(\omega t + kx)} \tag{161}$$

and $\hat{u}_n$ is the corresponding normal velocity amplitude (that also may be complex) for the same frequency component of u_n.

$$\hat{u}_n(x,\omega) = u_0 e^{i(\omega t + kx)} \tag{162}$$

The surface can be porous or rigid and its impedance depends on the frequency ω (in rad/s). So, $R(\omega)$ and $\alpha(\omega)$ are frequency dependent. For the reader's convenience, the variables t, ω and x are omitted from here on.

Due to the reflection of the sample at the end of the tube, the sound pressure can be written as:

$$\hat{p} = p_o(e^{ikx} + R\, e^{-ikx})e^{i\omega t} \tag{163}$$

With p_o the frequency dependent amplitude of the pressure, $k=\omega/c_0$ the wave number with c_0 the undisturbed speed of sound and R the reflection coefficient. $e^{i(\omega t+kx)}$ represents the forward travelling wave and $R\, e^{i(\omega t-kx)}$ the reflected wave. From the equation of momentum conservation (see chapter 2) the particle velocity can be derived:

$$\hat{u}_n = \frac{p_0}{\rho_0 c}(e^{ikx} - R \;\; e^{-ikx})e^{i\omega t} \tag{164}$$

Impedance tube

The reflection coefficient R is a complex figure that may be written as $R = |R|e^{i\nu}$ with θ the phase of the reflection coefficient. The Fourier transformed sound pressure and particle velocity in the tube are given by:

$$p = p_0(e^{ikx} + |R|e^{i\theta}\, e^{-ikx})$$
$$u = \frac{p_0}{\rho c}(e^{ikx} - |R|e^{i\theta}\, e^{-ikx}) \qquad (165)$$
$$u' = p_0(e^{ikx} - |R|e^{i\theta}\, e^{-ikx})$$

u' is the normalised particle velocity $u'=u\rho c$. The auto correlation functions leading to the auto spectral density or auto spectrum of the sound pressure and particle velocity in the tube are given by:

$$S_{pp} = \lim_{T\to\infty}\int_0^T \hat{p}^2 dt = \tfrac{1}{2}\, p\cdot p^* = \tfrac{1}{2}|p|^2 = \tfrac{1}{2}\, p_0^2\left(1 + |R|^2 + 2|R|cos(2kx+\theta)\right)$$
$$S_{uu} = \lim_{T\to\infty}\int_0^T \hat{u}^2 dt = \tfrac{1}{2}u\cdot u^* = \tfrac{1}{2}|\hat{u}|^2 = \frac{1}{2}\frac{p_0^2}{(\rho c)^2}\left(1 + |R|^2 - 2|R|cos(2kx+\theta)\right) \qquad (166)$$
$$S_{u'u'} = (\rho c)^2 S_{uu} = \frac{1}{2}p_0^2 \quad \left(1 + |R|^2 - 2|R|cos(2kx+\theta)\right)$$

The complex intensity $C(x)$ is given by:

$$C = \bar{I}_a + i\, I_{re} = \lim_{T\to\infty}\int_0^T \hat{p}(t)\cdot\hat{u}(t)dt$$
$$= \tfrac{1}{2}Re\{\, p\cdot u^*\,\} + \tfrac{1}{2}i\, Im\{\, p\cdot u^*\,\}$$
$$= Re\{\, S_{pu}\,\} + i\, Im\{\, S_{pu}\,\} \qquad (167)$$
$$= \sqrt{S_{uu}S_{pp}}\, cos\,\varphi_{pu} + i\sqrt{S_{uu}S_{pp}}\, sin\varphi_{pu}$$
$$= \frac{p_0^2}{2\rho c}(1 - |R|^2 - 2i|R|\, sin(2kx-\theta))$$

For practical situations it is convenient to redefine the spectral densities as single sided functions of frequency. The densities S in chapter C are therefore equal to the single sided densities G in chapter B.

The specific acoustic impedance in the tube is given by:

$$Z(x) = \frac{p}{u} = \rho c\frac{(e^{ikx} + R\, e^{-ikx})}{(e^{ikx} - R\;\; e^{-ikx})} = \frac{S_{pu}}{S_{uu}} \qquad (168)$$

At x=0 the impedance is simply given by:

$$\frac{Z(0)}{\rho_0 c} = \frac{1+R}{1-R} \tag{169}$$

As can be seen, the specific acoustic impedance at the end of the tube (at the sample) and the reflection coefficient of the sample are related in a simple manner.

The phase between the sound pressure and the particle velocity φ_{pu} is given by:

$$\begin{aligned}\varphi_{pu} &= Arg\{C\} = Arg\{Z\} = Arg\{S_{pu}\} \\ &= tg^{-1}\frac{Im\{S_{pu}\}}{Re\{S_{pu}\}} = tg^{-1}\frac{2\cdot|R|}{1-|R|^2}sin(\theta-2kx)\end{aligned} \tag{170}$$

Sound energy density is defined as the sum of potential energy U and the kinetic energy K per unit volume [39]. The potential energy is associated with the sound pressure and is given by:

$$U = \frac{1}{2\rho_0 c^2}\lim_{T\to\infty}\int_0^T \hat{p}(t)^2\,dt = \frac{|p|^2}{4\rho_0 c^2} = \frac{S_{pp}}{2\rho_0 c^2} \tag{171}$$

The kinetic energy K is associated with the particle velocity and is given by:

$$K = \tfrac{1}{2}\rho_0 \lim_{T\to\infty}\int_0^T \hat{u}(t)^2\,dt = \tfrac{1}{4}\rho_0|u|^2 = \tfrac{1}{2}\rho_0 S_{uu} \tag{172}$$

The sound energy density in the tube is given by:

$$\begin{aligned}E &= U + K = \frac{S_{pp}}{2\rho c^2} + \frac{\rho S_{uu}}{2} \\ &= \frac{S_{pp}}{2\rho c^2} + \frac{S_{u'u'}}{2\rho c^2} = \frac{1}{2}\frac{p_0^2}{\rho c^2}\left(1+|R|^2\right)\end{aligned} \tag{173}$$

With ρc the characteristic acoustic impedance and ρ the mean density of the medium (air). Sound energy density is, just as the sound intensity, place independent and therefore constant along the tube.

The p/u method

If particle velocity and sound pressure probes are at hand, the most obvious choice of determining the reflection coefficient of the sample is by simply measuring the specific acoustic impedance, the ratio of both acoustic quantities, see Eq. (168). As can be seen, this relation between the impedance at any place in the tube and the reflection coefficient of the sample is uniquely related. Until the introduction of the Microflown, the determination of the particle velocity on one place was not possible and maybe due to this the p/u method has never been developed.

The method is introduced here to get a proper overview. Sound pressure and particle velocity are measured at the same position L in the tube. The transfer function is given by the ratio between the signals of the pressure sensor and the Microflown:

$$H_{p/u} = \frac{\hat{p}(L)}{\hat{u}_x(L)} = \frac{p_0(e^{ikL}+Re^{-ikL})}{\frac{p_0}{\rho_0 c}(e^{ikL}-Re^{-ikL})} = \rho_0 c\frac{(e^{ikL}+Re^{-ikL})}{(e^{ikL}-Re^{-ikL})} = \frac{S_{pu}}{S_{uu}} \quad (174)$$

S_{pu} is the cross-spectrum and S_{uu} Auto-spectrum. Rewriting Eq. (174) results in an expression for the reflection coefficient at the sample material (x=0):

$$R = \frac{(H_{p/u}-1)}{(H_{p/u}+1)}\frac{e^{ikL}}{e^{-ikL}} = \frac{(H_{p/u}-1)}{(H_{p/u}+1)}e^{i2kL} \quad (175)$$

So, the reflection coefficient at the sample material is only a function of the transfer function $H_{p/u}$ and the distance L from the sample to the microphones. See also in chapter A, Fig. 35: "*Measurement in a standing wave tube: the sound pressure p, particle velocity u and the specific acoustic impedance (the ratio of both p/u)*".

The p-p method

For sound intensity measurement, the absence of a reliable velocity measurement in one place was answered by determining the velocity from the pressure difference that was obtained using two pressure microphones. Although the 2p method is normally used, the historically most obvious way of determining the specific acoustic impedance in the tube would be analogously performed by the use of two pressure

microphones. Similar to the sound intensity method the particle velocity calculation is done by measuring the pressure gradient. The method is discussed below.

With the use of two closely spaced pressure sensors, the particle velocity ***u*** can be determined (from the equation of momentum conservation), see chapter 2, Eq. (2.10):

$$u(x,t) = -\frac{1}{\rho_0}\int \frac{d}{dx}\, p(x,t)dt \approx -\frac{1}{\Delta x \cdot \rho_0}\int p(x+\Delta x,t) - p(x,t)dt \qquad (176)$$

Hence, determination of the impedance *Z(L)* at distance *x=L* in the so-called p-p-method follows from:

$$Z(L) = \frac{p(L)}{u(L)} \approx \frac{(p(L+\tfrac{1}{2}s) + p(L-\tfrac{1}{2}s))/2}{\frac{1}{\rho s}\int p(L+\tfrac{1}{2}s) - p(L-\tfrac{1}{2}s)dt} \qquad \left(= H_{p-p}\right)$$

$$= \frac{\rho s}{2}\frac{e^{ik(L+\tfrac{1}{2}s)} + e^{ik(L-\tfrac{1}{2}s)} + R\left[e^{-ik(L+\tfrac{1}{2}s)} + e^{-ik(L-\tfrac{1}{2}s)}\right]}{\frac{1}{i\omega}\left[\left(e^{ik(L+\tfrac{1}{2}s)} - e^{ik(L-\tfrac{1}{2}s)} + R\left[e^{-ik(L+\tfrac{1}{2}s)} - e^{-ik(L-\tfrac{1}{2}s)}\right]\right]} \qquad (177)$$

The transfer function $H_{p\text{-}p}$ and the reflection coefficient of the sample are uniquely related. So by measuring this transfer function, the reflection coefficient can be measured. In a similar way one could derive the **u-u** method.

Taking the limit s↓0, the transfer function $H_{p\text{-}p}$ will transform in $H_{p/u}$ (=*Z(L)*).

$$\lim_{s\to 0} H_{p-p} = \lim_{s\to 0}\frac{\rho s}{2}\frac{e^{ik(L+\tfrac{1}{2}s)} + e^{ik(L-\tfrac{1}{2}s)} + R\left[e^{-ik(L+\tfrac{1}{2}s)} + e^{-ik(L-\tfrac{1}{2}s)}\right]}{\frac{1}{i\omega}\left[\left(e^{ik(L+\tfrac{1}{2}s)} - e^{ik(L-\tfrac{1}{2}s)} + R\left[e^{-ik(L+\tfrac{1}{2}s)} - e^{-ik(L-\tfrac{1}{2}s)}\right]\right]} \qquad (178)$$

$$= \rho c\frac{e^{ikL} + R\,e^{-ikL}}{e^{ikL} - R\,e^{-ikL}} = H_{p/u}$$

So in for certain bandwidth and microphone spacing *s*, the $H_{p\text{-}p}$ method and the $H_{p/\boldsymbol{u}}$, method are mathematically similar.

The 2p and 2u method

The most common way nowadays to determine the specific acoustic impedance in the tube is (instead of the p-p method) the 2p method [33]. In this method the ratio of two pressure microphones used to determine the properties of the sample. The similar 2**u** method is explained as well in this paragraph.

By measuring the velocity at two distinct points in the tube the transfer function, $H_{2u}=u_2/u_1$, can be calculated [40]. From the transfer function H_{2u} the reflection coefficient of the sample material can be derived. The two positions in the tube where the velocity is measured are given in Fig. 100, at a distance $x=L$ and at a distance $x=L-s$ from the sample.

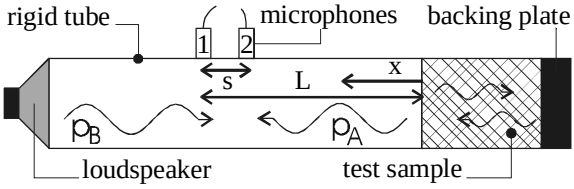

Fig. 100: Outline of an impedance tube.

$$H_{2u} = \frac{u_2}{u_1} = \frac{e^{ik(L-s)} - R\,e^{-ik(L-s)}}{e^{ikL} - R\,e^{-ikL}} = \frac{S_{u_1u_2}}{S_{u_1u_1}} \tag{179}$$

S_{u1u2} is the cross-spectrum of both probes and S_{u1u1} the auto-spectrum of probe $\boldsymbol{u}_1$. Rewriting Eq. (179) results in an expression for the reflection coefficient at the sample material (x=0):

$$R = \frac{e^{ik(L-s)} - H_{2u}e^{ikL}}{e^{-ik(L-s)} - H_{2u}e^{-ikL}} \tag{180}$$

Analogously to the ratio of the velocities measured at $x=L$ and $x=L-s$, it is possible to define a quotient of p_1 and p_2, yielding

$$H_{2p} = \frac{p_2}{p_1} = \frac{e^{ik(L-s)} + R\,e^{-ik(L-s)}}{e^{ikL} + R\,e^{-ikL}} = \frac{S_{p_1p_2}}{S_{p_1p_1}} \tag{181}$$

where p_1 and p_2 represent the measured pressures at $x=L$ and $x=L-s$.

Applying some mathematics, it can be deduced that there is a mutual and unique relation between H_{2u} and H_{2p}, i.e.

$$H_{2u} = \frac{(1+e^{2iks})H_{2p} - 2e^{iks}}{2e^{iks}H_{2p} - (1+e^{2iks})} \tag{182}$$

Expression (182) shows that, since H_{2u} and H_{2p} are uniquely related and only the spacing s (not even L) appears, the 2u and 2p-method are equivalent and not fundamentally different. From the viewpoint of determining the ratio H, both methods can equally adequately be used. Fig. 101 shows the measured signals such as defined in Eq. (179) and (181) for an acoustic hard wall. As can be seen the signals are quite similar and have a dynamic range of about 10,000 (80dB).

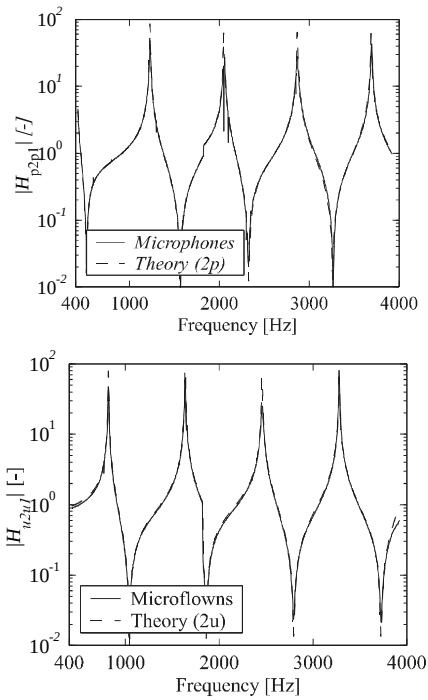

Fig. 101: Transfer function for an acoustic hard wall using the 2p and the 2u method.

Impedance tube

The signals that are displayed in Fig. 101 are used to calculate the reflection coefficient that should be unity (acoustic hard wall). The result is shown in Fig. 102.

For both methods the experimental results correspond to the theoretical reflection coefficient of $|R|=1$, except for the higher frequencies some deviations can be seen. A reflection coefficient $|R|>1$ indicates inaccuracies in the measurements for higher frequencies. The calculation of R for a hard wall is relatively sensitive to these inaccuracies. This is explained by the sensitive transformation of the measured transfer function, which ranges theoretically from $+\infty$ to $-\infty$, to the reflection coefficient of $R=1$. This causes numerical problems and incorrect values of the impedance.

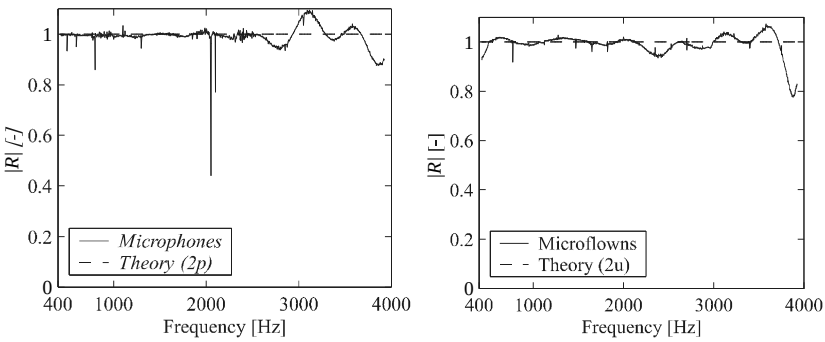

Fig. 102: Reflection coefficient of an acoustic hard wall using the 2p and the 2u method.

For small microphone spacing s, the p-p-method, provide similar information as p/u-method. Besides, the determined impedance Z is also uniquely related to the transfer functions H_{2u} and H_{2p}. Calculations analogue to those at Eq. (182), lead to:

$$H_{p/u} = \rho_0 c \frac{(e^{2iks}-1)H_{2p}}{1+e^{2iks}-e^{iks}H_{2p}} \tag{183}$$

From this relationship it can be concluded that the determination of Z(0) at the sample, either measured with the p/u-method or with the p-p-method, is similar to, and gives the same information as, the method of the 2p-method from which H_{2p} is found. H_{2u} and H_{p2} are uniquely,

relatively simply, related. The methods only differ in a practical manner: it can for example be advantageous to measure the acoustic impedance in one spot (p/u method), similarity of Microflowns is high (2**u** method) compared to low cost microphones (2p method).

The p·u method

In the previous paragraph it is shown that in fact all the methods (2p, p-p, **u-u,** p-p and p/**u**) are mathematically the same. They are all determining some sort of impedance to be able to calculate the reflection coefficient at the end of the tube.

This p-**u** method is determining the reflection coefficient with a slightly different approach. Imagine the sample in the tube is full sound reflecting: all the sound that is put into the tube will be reflected at the end. The sound intensity (the net flow of sound energy in one direction) in the tube will be zero due to this. If on the other hand the sample is fully sound absorbing, the sound intensity will be a large figure. How large this figure is depends (apart from the absorption) on the amount of noise that is generated by the loudspeaker. The loudness can be determined by other acoustic quantity, the sound energy density. The ratio of the intensity and the energy is zero for a full reflecting sample (intensity is zero) and (apparently) a half the speed of sound for a sample that is full absorbing.

This expression is independent on the place but, however, dependent on the driving sound pressure of the loudspeaker. To be able to determine the amount of sound that is generated by the loudspeaker, an alternative method therefore is utilised based on the so-called sound energy density *E*.

The ratio of the sound intensity and the sound energy density in the tube is given by:

$$\frac{\overline{I}_a}{E} = \frac{Re\left\{S_{pu}\right\}}{\dfrac{S_{pp}}{2\rho_0 c^2} + \dfrac{1}{2}\rho_0 S_{uu}} = c \cdot \frac{1-|R|^2}{1+|R|^2} \tag{184}$$

Eq. (184) is quite similar to Eq. (174). The difference is that in Eq. (184) the real part of the cross spectrum is used and that in Eq. (174) (apart from some constants) only the kinetic energy is used. The advantage of using apart from the kinetic energy also the potential energy is that the denominator will not become zero. This will avoid numerical

problems. The research is still in an exploratory stage [41]. Some measurements and theory is presented below.

The method that is described with Eq. (184) results in the absolute value for the reflection coefficient only. The ratio of the imaginary part of the cross spectrum of p and **u** (S_{pu}) and the sound energy contains information on the phase of the reflection coefficient. The ratio of the complex intensity and energy in the tube is given by:

$$\frac{C}{E} = 2\frac{S_{pu}}{\frac{S_{pp}}{\rho c^2} + \rho_0 S_{uu}} = 2c\frac{\sqrt{S_{u'u'}S_{pp}}\cos\varphi_{pu} + i\sqrt{S_{u'u'}S_{pp}}\sin\varphi_{pu}}{S_{pp} + S_{u'u'}}$$
$$= c\left(\frac{1-|R|^2}{1+|R|^2}\right) - 2ic\left(\frac{|R|}{1+|R|^2}\right)\sin(2kx-\theta) \tag{185}$$

The modulus of the reflection coefficient is given by:

$$|R|^2 = \frac{cE - I_a}{cE + I_a} = \frac{\frac{1}{2}S_{u'u'} + \frac{1}{2}S_{pp} - \sqrt{S_{u'u'}S_{pp}}\cos\varphi_{pu}}{\frac{1}{2}S_{u'u'} + \frac{1}{2}S_{pp} + \sqrt{S_{u'u'}S_{pp}}\cos\varphi_{pu}} \tag{186}$$

The phase of the reflection coefficient is given by:

$$\theta = \sin^{-1}\left(\frac{1-|R|^2}{2|R|}tg\varphi_{pu}\right) + \frac{4\pi}{c}f\cdot x \tag{187}$$

Experimental set-up

The impedance tube is sketched in Fig. 103. Sound pressures p1 and p2 indicate the microphone positions for the 2p method. The same positions are used for the 2u method but then two Microflowns are used. For the p/**u** and the p·**u** method, p and u indicate the positions of the microphone and the Microflown, respectively. Notice that for the latter two methods there is no separation distance *s* and that the distance to the sample *L* is different (*L* = 0.407 m). The microphones for the 2p method are ¼ inch Kulites. For the p/u and p·**u** method a ½ inch B&K microphone and a ½ inch ICP Microflown Technologies is used.

Experimental results

Before the reflection coefficient R (or the sound absorption coefficient α) of a sample with a quarter-wave resonator is measured the sensors are calibrated. The procedure and the calibration results for the combination of a microphone and a Microflown are described in chapter A5 "*Calibration methods*". The measured and theoretical transfer function $H_{u/p}$ are given in Fig. 104. A minimum occurs when the velocity has a minimum value at the location of the Microflown.

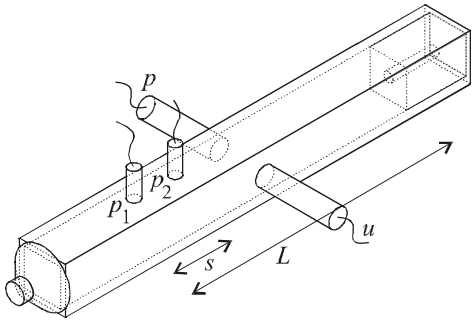

Fig. 103: Impedance tube and sensor positions.

Calibration results

From the measured and theoretical values (theory: without gain) a correction function is obtained as a function of the frequency. The difference for the amplitude and the phase is given in the Fig. 105. As a first approach a third order polynomial is used for the final interpolated correction. In chapter B2: "*a p**u** intensity measurement*" the phase response is fitted more successfully with a inverse tangent function.

Impedance tube

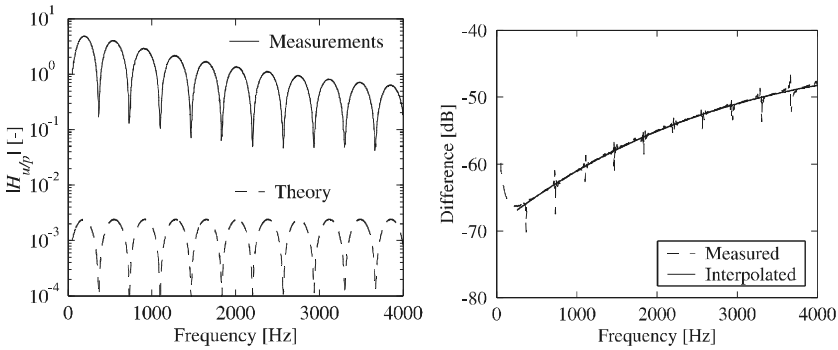

Fig. 104: Left: transfer function $H_{u/p}$ (magnitude). Right: amplitude difference in dB.

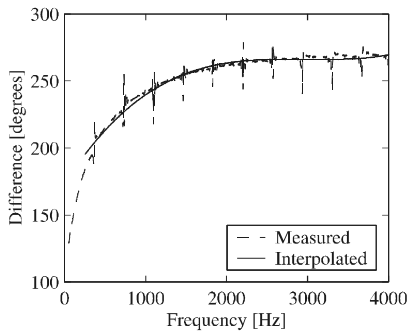

Fig. 105: Phase difference in degrees, the solid line is the approximation that is used for the calculations.

In Fig. 105 one can see that the estimation that is used for the calculations (the solid line) is not very accurate. This will lead to a miscalculation in the sound intensity estimation.

For the present tests the measured transfer function $H_{u/p}$ is corrected in the following way:

$$20\log\left|\tilde{H}_{u/p}\right| \rightarrow 20\log\left|H_{u/p}\right| + \text{Correction} \quad [\text{dB}]$$

$$\arg(\tilde{H}_{u/p}) \rightarrow \arg(H_{u/p}) + \text{Correction} \qquad [\text{deg}]$$

PU probe calibration results

The p**u**-probe that is described in chapter B2: "*a p**u** intensity measurement*" the frequency and phase response of both sound probes are calibrated in a standing wave tube.

Models to correct for the amplitude response of the Microflown and microphone are given by:

$$H_u = 20Log\frac{2.35}{\sqrt{\left(\frac{f^2}{620^2}+1\right)\left(\frac{f^2}{2500^2}+1\right)}} \quad ; \quad H_p = 20Log1.5\sqrt{\left(\frac{f^2}{6800}+1\right)\left(\frac{f^2}{7800}+1\right)} \qquad (188)$$

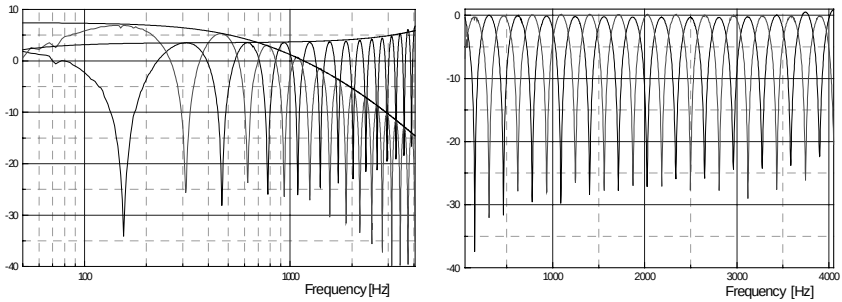

Fig. 106: Left: frequency response of a p**u**-probe in a standing wave tube, the solid line is the approximation that is used for the calculations. Right the corrected response.

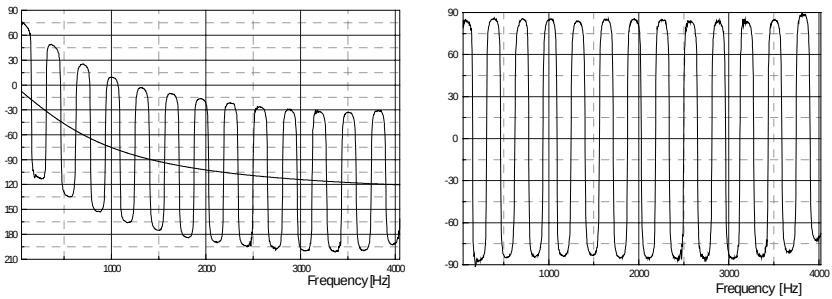

Fig. 107: Left: phase response of a p**u**-probe in a standing wave tube, the solid line is the approximation that is used for the calculations. Right the corrected response.

The model that is used to correct for the phase response of the Microflown is given by:

$$\varphi_{corr} = -1.52 \cdot tg^{-1}\left(\frac{f}{914}\right) - 3 \tag{189}$$

Absorption measurements

The measurement set-up is depicted in Fig. 103 and the results are presented in Fig. 108, Fig. 109 and Fig. 110. For the results with the 2u method reference is made to [8] where it is shown that the 2u method provides the same results as the 2p method. A special sample has been used. It is a so called “quarter-wave resonator” and a specific frequency the sample has a high damping. Details of this resonator can be found at [41] and [10].

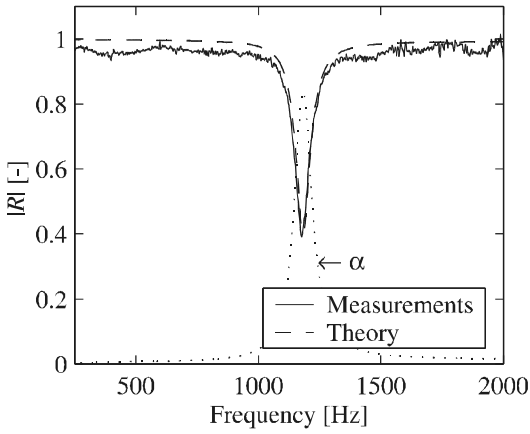

Fig. 108: Reflection coefficient (magnitude) using the 2p method.

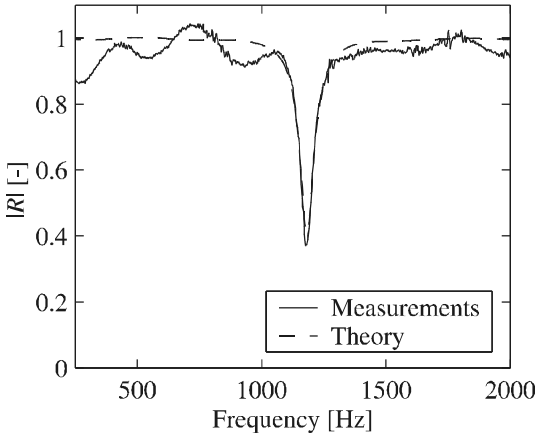

Fig. 109: Reflection coefficient (magnitude) using the p/u method.

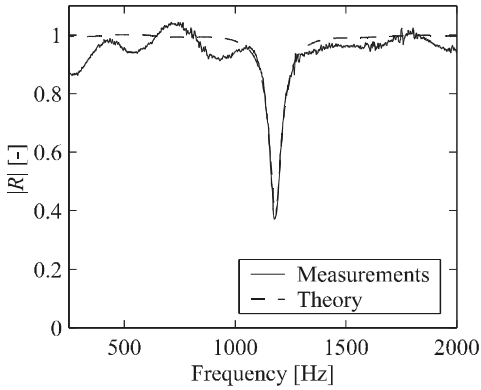

Fig. 110: Reflection coefficient (magnitude) using the p·u method.

The figures show that the three methods provide identical results for the frequency range where the quarter-wave resonator is effective, i.e. from 1000 to 1400 Hz. For the lower frequency range the **u**/p and the p-**u** method are less accurate. This may be improved with a more accurate calibration of the transfer function $H_{u/p}$. It is noted that the phase-calibration can be performed with the microphone flush mounted. In this way the microphone shows the same phase-response behaviour because

the incident sound field is the same as for the experiments with a sound absorbing sample. In this respect it has to be remarked that the calculation of *R* for an acoustic hard wall is very sensitive to measurement inaccuracies.

PU method: some signals

The velocity and pressure response due to a quarter lambda resonator that is positioned at the end of a standing wave tube are depicted below.

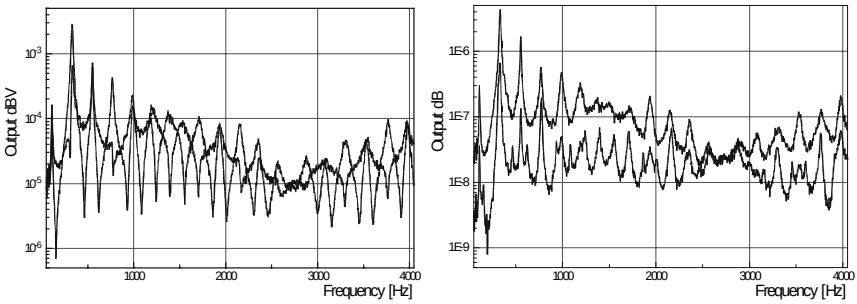

Fig. 111: Left the auto spectra of both Microflown as microphone signal: quite large dynamic signals. Right: the speed of sound times energy (upper) and active intensity (lower curve). Dynamic range of both signals is not very large. As can be seen at 2500Hz-30kHz, if $cE=I_a$ the reflection coefficient is small, see also Fig. 112 right.

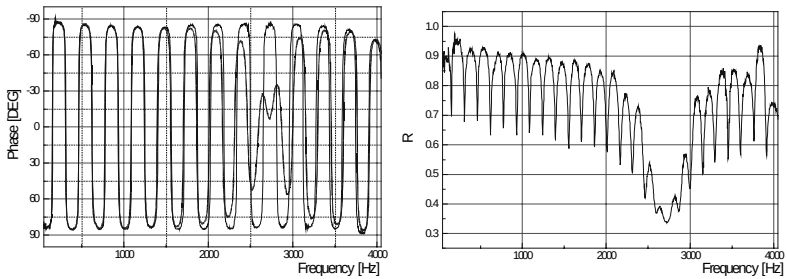

Fig. 112: Left the phase response between pressure and velocity in the tube for R=1 (thin line varying almost =90 to –90 degrees) and thick line response of when a quarter lambda resonator is positioned at the end of the tube. Right: the calculated reflection coefficient of a quarter lambda resonator is positioned at the end of the tube (other damping effects of the tube are still visible).

Transmission measurement set-up

This method is not tested yet. However the method looks promising and therefore it will be presented here. If the sample is put at the end of the tube, the situation alters as shown in Fig. 113.

Fig. 113: Set-up for measuring the transmission coefficient.

Since the tube is terminated rigidly, in the second part of the tube all sound is reflected. All formulas that are given in the previous paragraph still apply for the part before the sample. Besides that, due to the full reflection at the end of the tube, the (active) sound intensity in the part behind the sample will be zero. The reactive sound intensity on the other hand is not zero. Because of its place dependence, measurement of the reactive part of the sound intensity is not convenient. The measurement of the sound energy density E is utilised again. Due to the damping of the sample the sound energy density in part before the sample is larger than in the part behind the sample.

The amount of sound that will be transmitted through the sample is quantified with the complex transmission coefficient T.

Simple mathematics shows that the sound energy density is place independent in both parts of the tube. The energy at the end of the tube can be determined by the sound pressure only because the velocity is zero at that position. So in fact the same set up that is used for the calibration can be used.

If a sound absorbing material is put in the tube, the amount of sound that will be transmitted through the sample is quantified with the transmission coefficient $T = |T| e^{\breve{}}$. The sound pressure and particle velocity behind the sample (*bs*) in the tube are given by:

$$p_{bs} = |T| e^{\delta} p_0 (e^{ikx} + e^{-ikx}) e^{i\omega t} \; ; \; u_{bs} = \frac{p_0}{\rho c} |T| e^{\delta} (e^{ikx} - e^{-ikx}) e^{i\omega t} \; ; \quad (190)$$

Impedance tube

The auto spectrum of the sound pressure and particle velocity in the tube are given by:

$$S_{pp,bs} = \tfrac{1}{2}\left|p_{bs}\right|^2 = \tfrac{1}{2}\,p_0^2\left(2\left|T\right|\cos kx\right)^2$$
$$S_{u'u',bs} = (\rho c)^2 S_{uu,bs} = \tfrac{1}{2}\,p_0^2\left(2\left|T\right|\sin kx\right)^2 \tag{191}$$

The sound energy density behind the sample is given by:

$$E_{bs} = \frac{p_0^2\left(\left|T\right|\cos kx\right)^2}{\rho c^2} + \frac{p_0^2\left(\left|T\right|\sin kx\right)^2}{\rho c^2} = \frac{p_0^2}{\rho c^2}\left|T\right|^2 \tag{192}$$

If the sound energy density is measured at the end of the tube, the particle velocity will be zero and it can be measured with use of a reference sound pressure microphone (p_r).

$$E_{bs}(x=0) = E_{bs,0} = \frac{S_{p_r p_r}}{2\rho c^2} = \frac{p_0^2}{\rho c^2}\left|T\right|^2 \tag{193}$$

The ratio of the sound energy densities in both parts of the tube is given by:

$$\frac{p}{p_{bs}} = \frac{e^{ikx} + \left|R\right|e^{i\theta}\,e^{-ikx}}{\left|T\right|e^{i\delta}\left(e^{ikx_{bs}} + e^{-ikx_{bs}}\right)} = \frac{S_{pp_{bs}}}{S_{p_{bs}p_{bs}}} \rightarrow \left|T\right|e^{i\delta} = \frac{e^{ikx} + R\,e^{-ikx}}{2\dfrac{S_{pp_{bs}}}{S_{p_{bs}p_{bs}}}\cos(kx_{bs})}$$
$$\frac{u'}{p_{bs}} = \frac{e^{ikx} - \left|R\right|e^{i\theta}\,e^{-ikx}}{\left|T\right|e^{i\delta}\left(e^{ikx_{bs}} + e^{-ikx_{bs}}\right)} = \frac{S_{u'p_{bs}}}{S_{p_{bs}p_{bs}}} \rightarrow \left|T\right|e^{i\delta} = \frac{e^{ikx} - R\,e^{-ikx}}{2\dfrac{S_{u'p_{bs}}}{S_{p_{bs}p_{bs}}}\cos(kx_{bs})} \tag{194}$$

The position of the pressure microphone (p_{bs}) behind the sample is at the end of the tube so at x=0 and therefore:

$$\left|T\right|e^{i\delta} = \frac{e^{ikx} + R\,e^{-ikx}}{2\dfrac{S_{pp_{bs}}}{S_{p_{bs}p_{bs}}}}\;;\;\left|T\right|e^{i\delta} = \frac{e^{ikx} - R\,e^{-ikx}}{2\dfrac{S_{u'p_{bs}}}{S_{p_{bs}p_{bs}}}} \tag{195}$$

The modulus of the transmission coefficient can be calculated by:

$$\frac{E}{E_{bs,0}} = \frac{S_{pp} + S_{u'u'}}{S_{p_{bs}p_{bs}}} = \frac{1}{2}\frac{\left(1+|R|^2\right)}{|T|^2} \rightarrow |T|^2 = \frac{\left(1+|R|^2\right)}{2}\frac{S_{p_{bs}p_{bs}}}{S_{pp} + S_{u'u'}} \qquad (196)$$

The phase of the transmission coefficient and the reflection coefficient are uniquely related because the absorption of the sample is a real figure.

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