Practical Parallel Programming V

Performance Analysis

See Chapter 6 of Jan's PhD

Kumar Chapter 5

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Performance Metrics

We assume sequential version is run on the same processor/core as the parallel version.

$$Speedup = \frac{T_{seq}}{T_{par}}$$

$$Efficiency = \frac{Speedup}{p} = \frac{T_{seq}}{p.T_{par}}$$

Other metrics we could try to optimize: energy consumption, cost, …

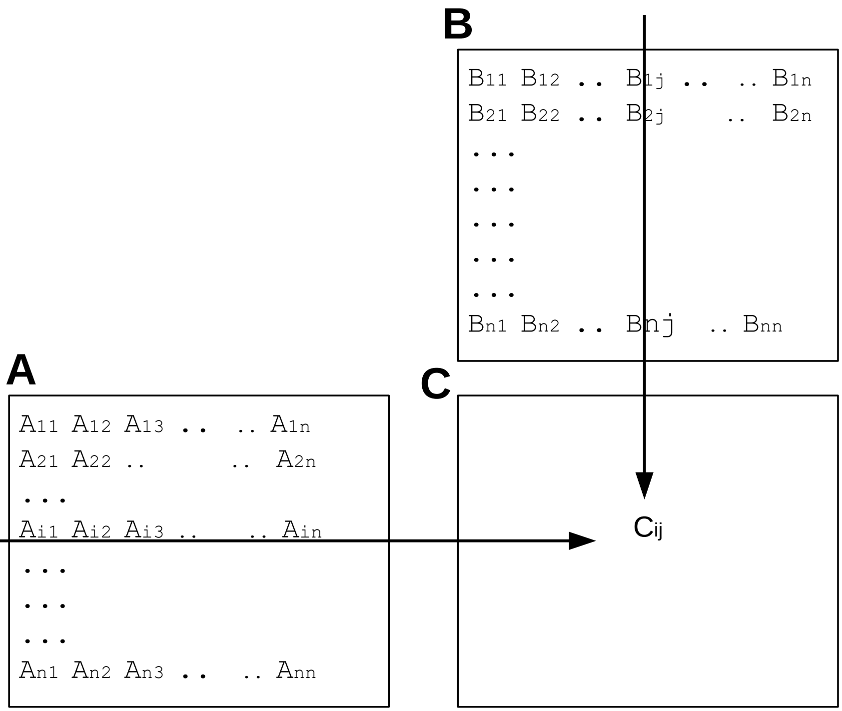
Matrix Multiplication (MxM)

$$C = A \times B$$

$$C_{ij} = \sum_{k=1}^{n} A_{ik} . B_{kj} \quad (i, j : 1..n)$$

$$T_s = \delta_{mm} . n^3$$

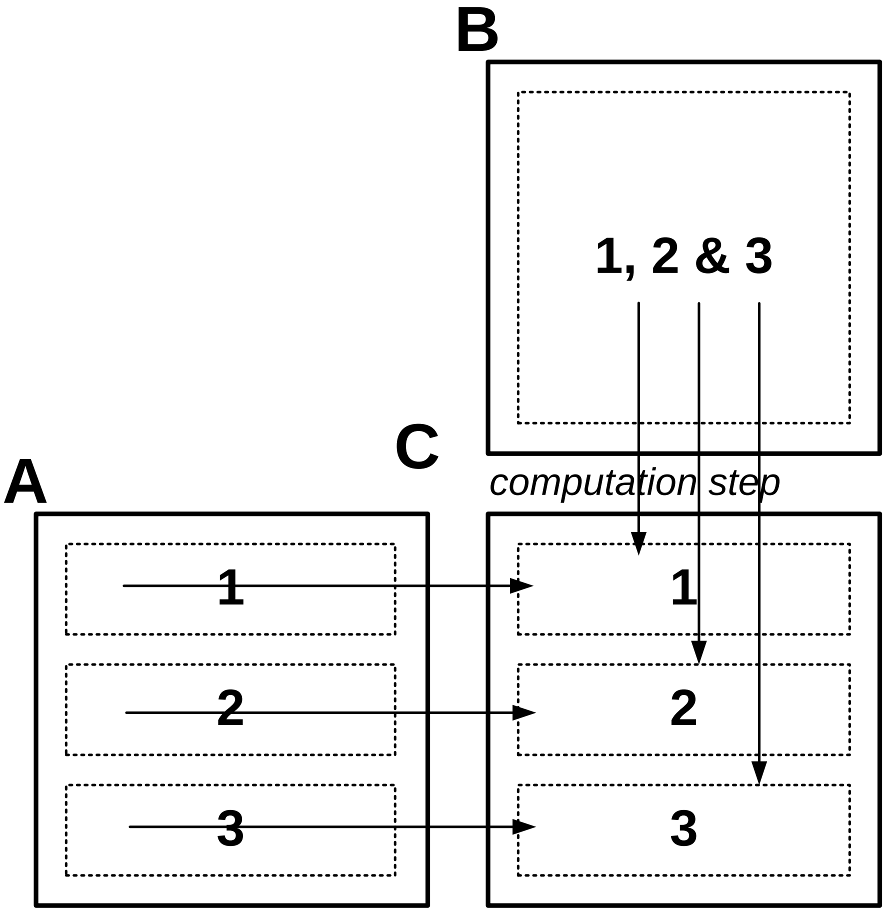
```
for (i=0; i<n; i++){
    for (j=0; j<n; j++){
        C[i,j]=0;
        for (k=0; k<n; k++){
            C[i,j]+=A[i, k]*B[k,j];
        }
    }
}
```



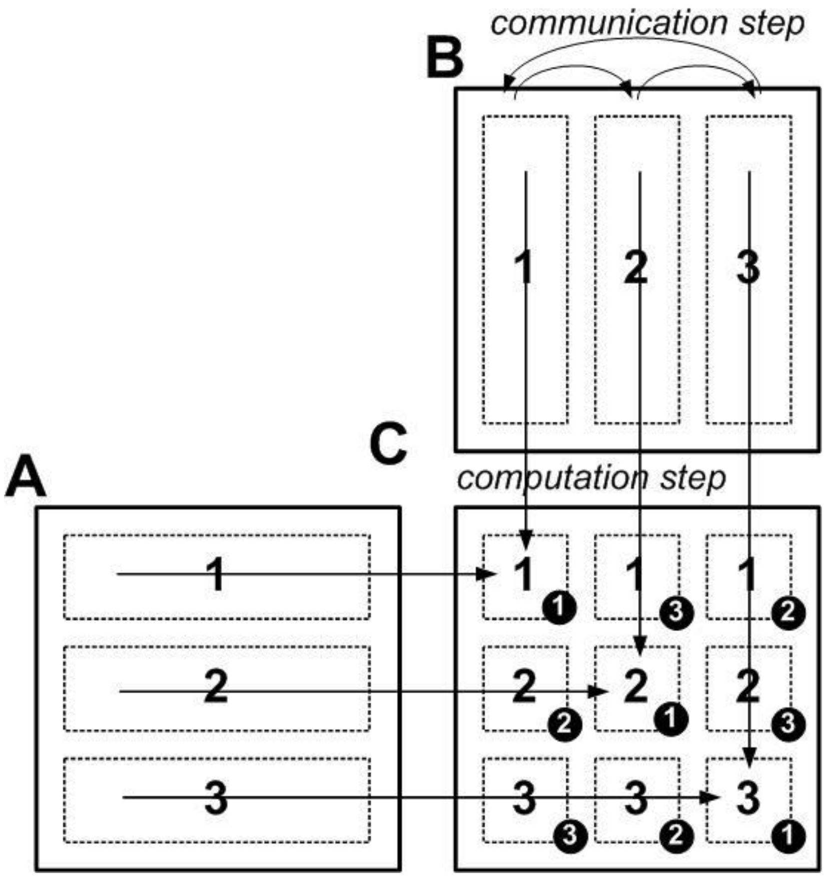
MxM: one-step version

- B is sent to all processors
- Computation in 1 step
- Same amount

of communication



MxM: Alternate shift-compute version

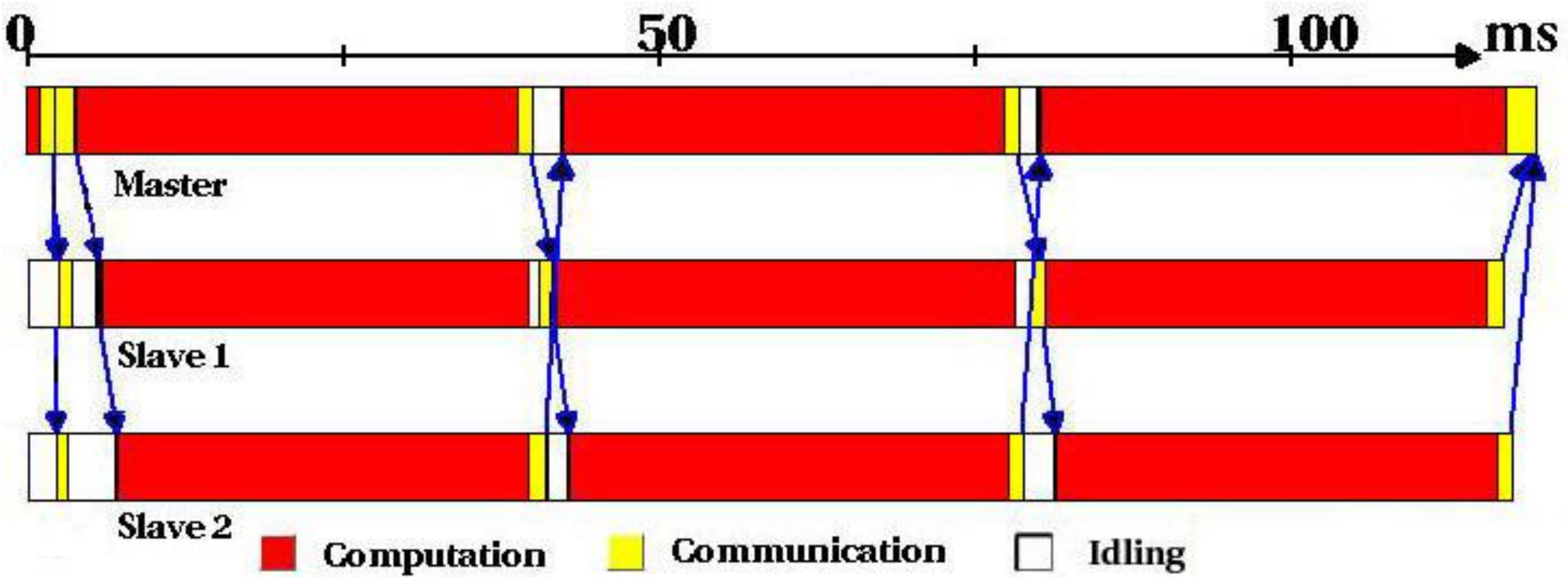


- Algorithm alternates p computation and communication steps
- Computation step: each processor multiplies its A submatrix with its B submatrix, resulting in a submatrix of C. The black circles indicate the step in which each submatrix is computed.
- After multiplication: processor sends it B submatrix to next processor and receives one from the preceding processor. The communication forms a **circular shift operation**.

Parallel Matrix Multiplication: Execution Profile

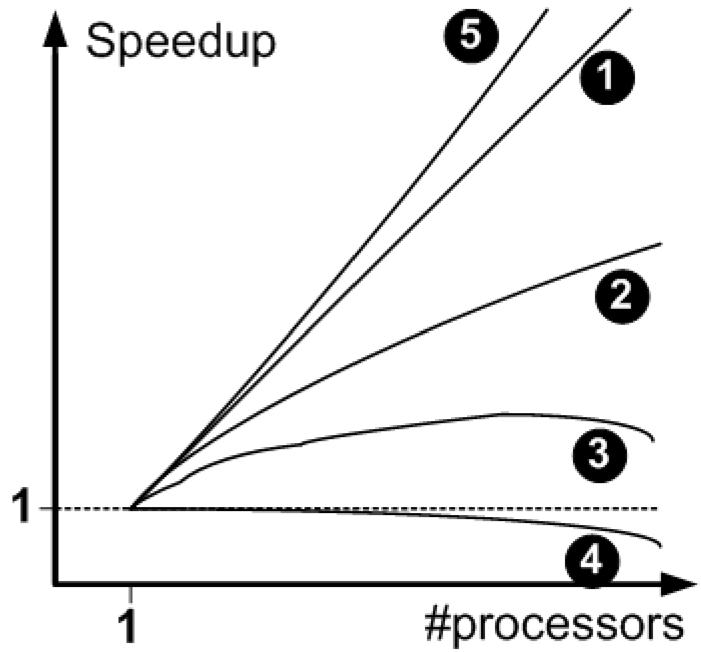
The alternate shift-compute version

On cluster of 3 computers - using MPI



Speedup=2.55 Efficiency = 85%

Speedup i.f.o. processors



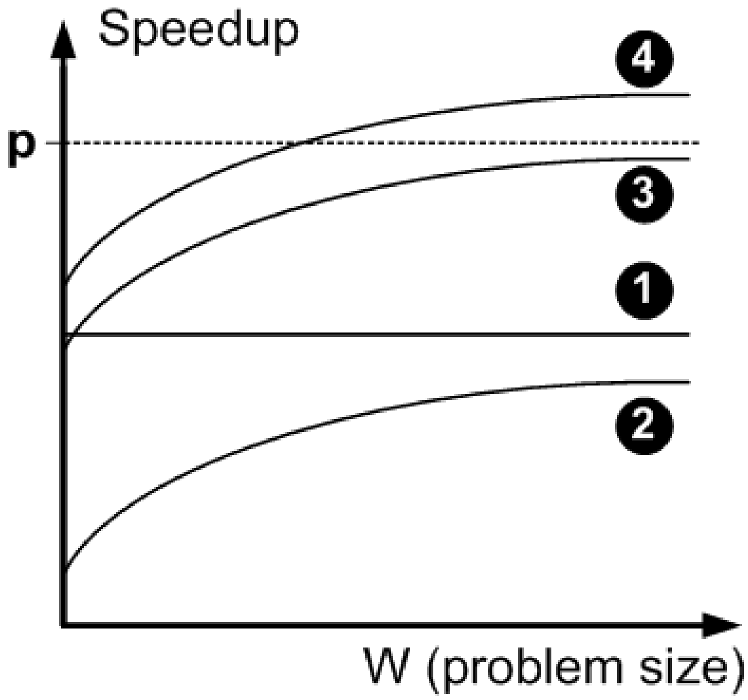
1) Ideal, linear speedup
2) Increasing, sub-linear speedup
3) Speedup with an optimal number of processors
4) No speedup
5) Super-linear speedup

PPP p. 92

Super-linear speedup

- The parallel execution works with data that fits in lower-level memory, while this is not the case for the sequential execution
- The work in parallel is less than that of the sequential program, called *parallel anomaly*.

Speedup i.f.o. problem size



1) Constant speedup
2) Increasing, asymptotically, towards value sublinear speedup (< *p*)
3) Increasing towards *p*
4) Increasing towards super-linear speedup

W is a problem-specific parameter which is related to the amount of computational work (most often linearly-related)

Performance Analysis

Goals:

- Understanding of the computational process in terms of resource consumption
- Identification of inefficient patterns
- Performance prediction
- Performance characterization of program and system

Overhead or *Lost Cycles*

Ideally $T_{par} = T_{seq}/p \Rightarrow Speedup = p$

In practice $T_{par} > T_{seq}/p$

$$overhead = p.T_{par} - T_{seq} \Rightarrow \boxed{lost\ processor\ cycles}$$

= all cycles with T_{par} that are not utilized for **useful work**

For all processes:

$$T^{i}_{par} = T^{i}_{work} + \sum_{j}^{O} T^{i,j}_{ovh}$$

i: index of process *j*: index of overhead type

Impact of Overhead on Speedup?

$$T_{seq} + T_{anomaly} = \sum_{i}^{p} T_{work}^{i}$$

$$T_{par} = T_{par}^{1} = \ldots = T_{par}^{p} \Rightarrow T_{par} = \frac{\sum_{i}^{p} T_{par}^{i}}{p}$$

$$T_{par} = \frac{\sum_{i}^{p} T_{work}^{i} + \sum_{i}^{p} \sum_{j}^{O} T_{ovh}^{i,j}}{p}$$

$$= \frac{T_{seq} + T_{anomaly} + \sum_{j}^{O} T_{ovh}^{j}}{p}$$

We sum overheads of type *j* over all processes *i*

$$Speedup = \frac{T_{seq}}{\frac{T_{seq} + \sum_{j}^{O+1} T_{ovh}^{j}}{p}} = \frac{p}{\frac{T_{seq}}{T_{seq}} + \frac{\sum_{j}^{O+1} T_{ovh}^{j}}{T_{seq}}}$$

We consider $T_{anomaly}$ an overhead

Speedup & Overhead Ratios

$$Speedup = \frac{p}{1 + \sum_j \frac{T^j_{ovh}}{T_{seq}}}$$

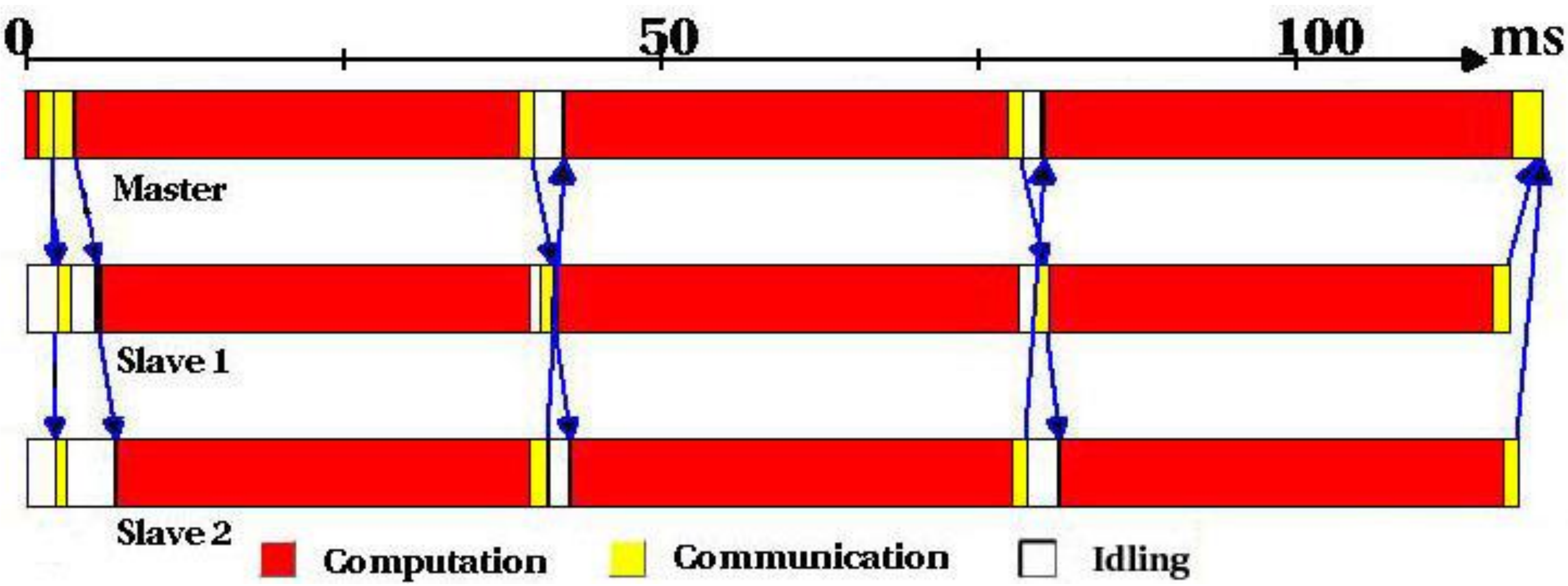
$$Ovh_j = \frac{T^j_{ovh}}{T_{seq}}$$

$$Speedup = \frac{p}{1 + \sum_j Ovh_j}$$

$$Efficiency = \frac{1}{1 + \sum_j Ovh_j}$$

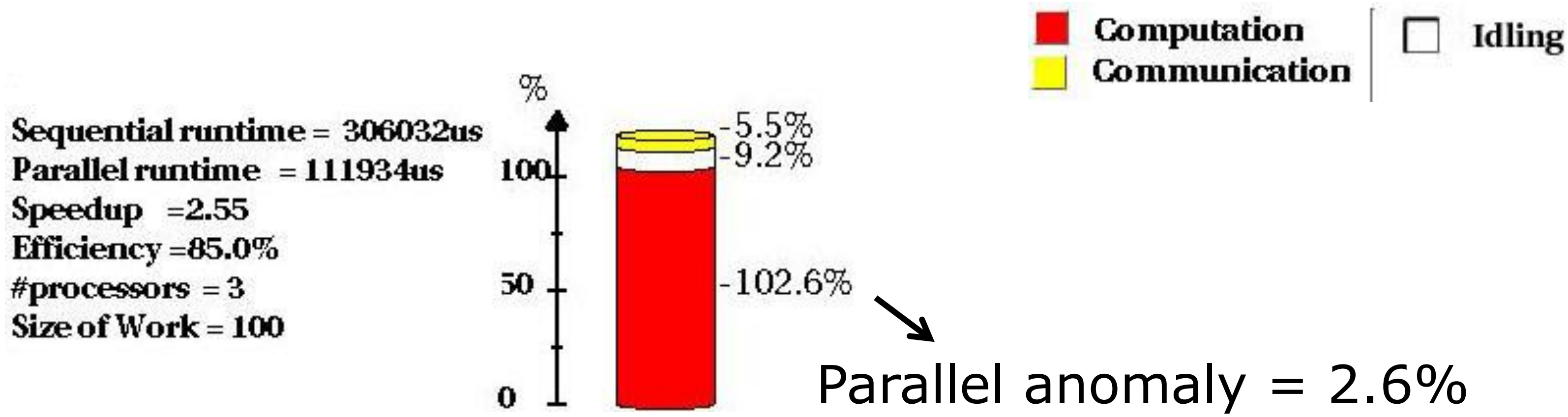
Total overhead (sum of per process overhead)!

Example 1: Execution Profile of Parallel Matrix Multiplication



Speedup=2.55 Efficiency = 85%

Parallel Matrix Multiplication



Parallel anomaly = 2.6%

$$\boxed{Efficiency = \frac{1}{1 + (5,5 + 9,2 + 2,6)/100} = \frac{1}{1,173} = 0,85}$$

Analysis per process

If you assume that each process has $\boxed{\dfrac{T_{seq}}{p}}$ work,

We can calculate the overhead ratio per process: $\boxed{Ovh_j^i = \dfrac{T_{ovh}^{i,j}}{T_{seq}/p}}$

Overhead Classification

- ***Control of parallelism***: extra functionality necessary for parallelization (like partitioning)
  - Extra computations required
  - Part of computational phases are not for useful work!

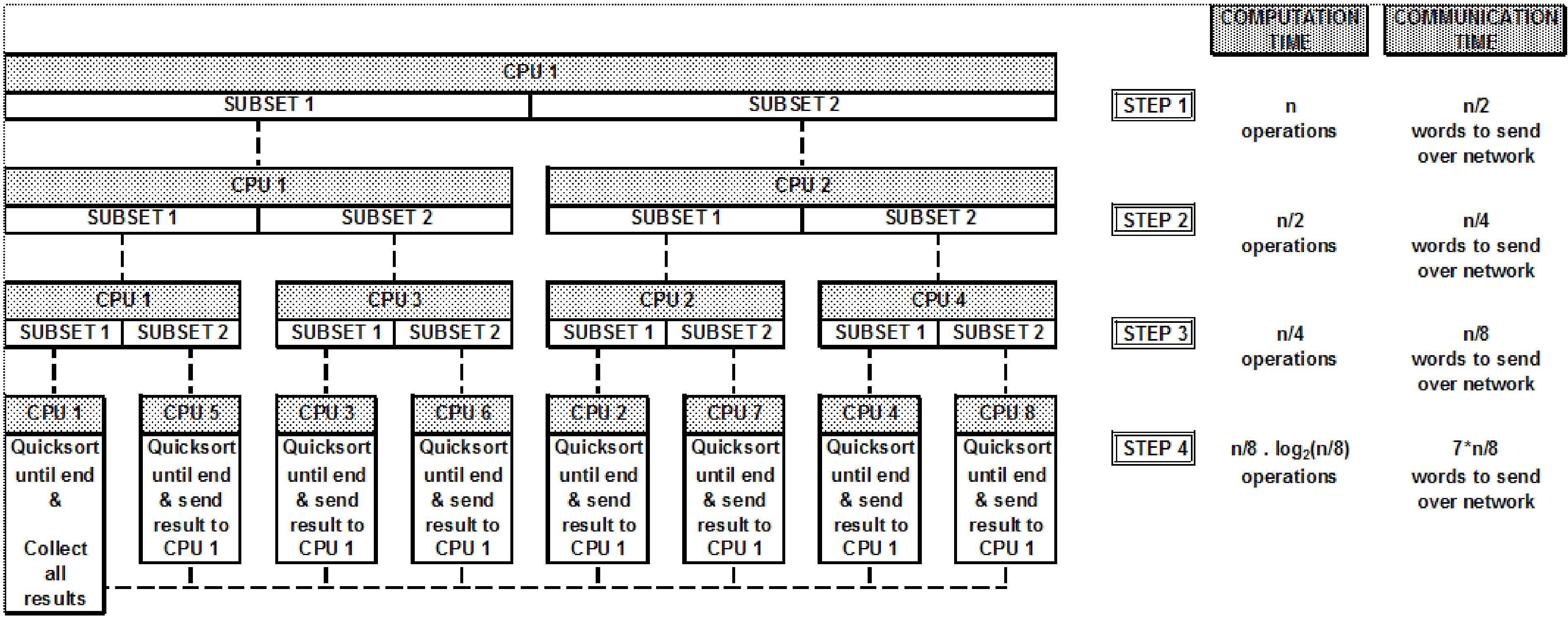
  Example of costly control: graph partitioning is NP-complete
- ***Communication***: overhead time not overlapping with computation
- ***Idling***: processor has to wait for further information
- ***Parallel anomaly*** : useful work differs for sequential an

$$T_{seq} + T_{anomaly} = \sum_{i}^{p} T^{i}_{work}$$

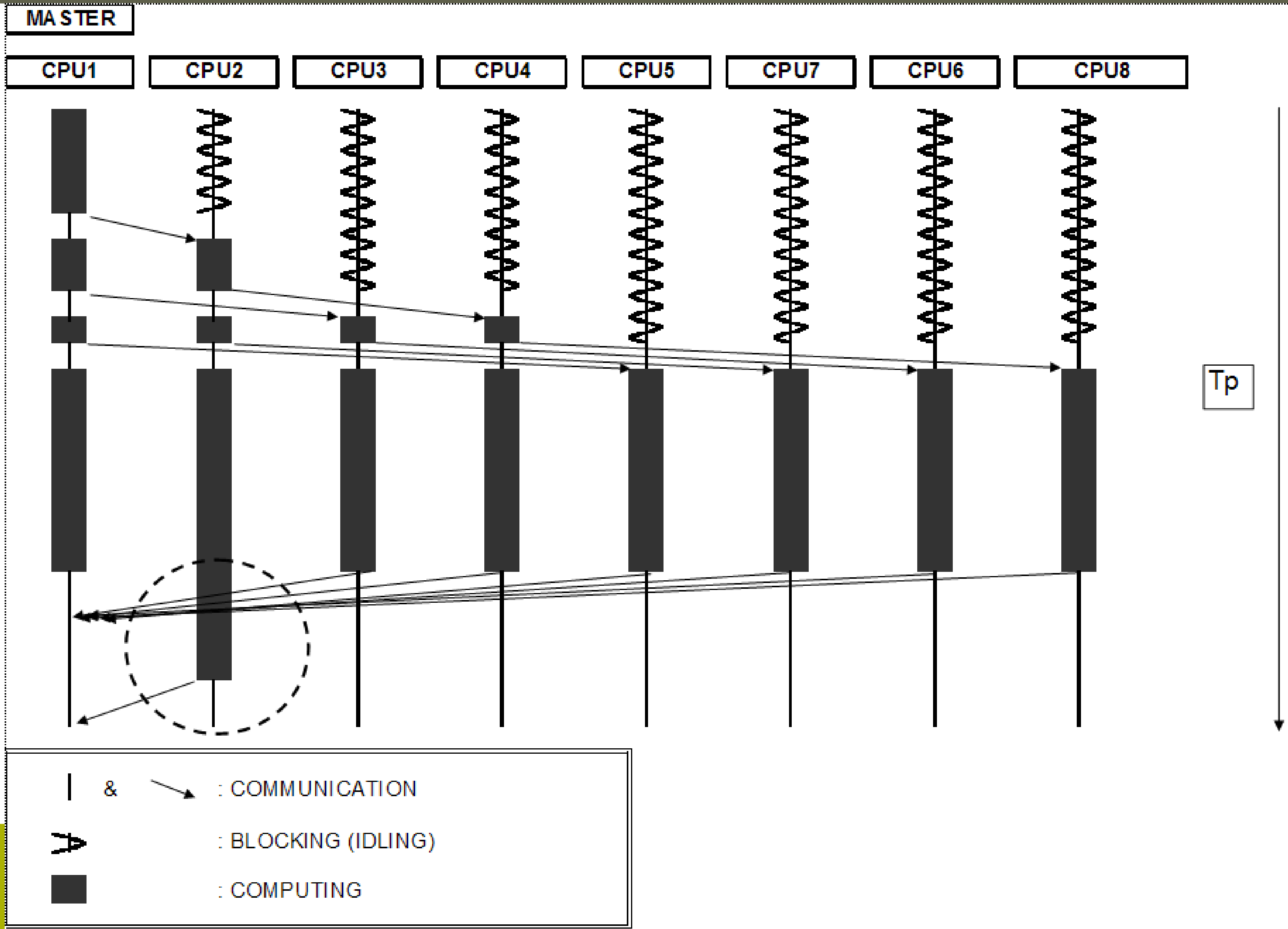
Causes of Idling

- Limitations of parallelism
  - Cf Amdahl's law
- Load imbalances
- Waiting for incoming messages, due to
  - Message latency
  - Limited bandwidth
  - Congestion in interconnection network

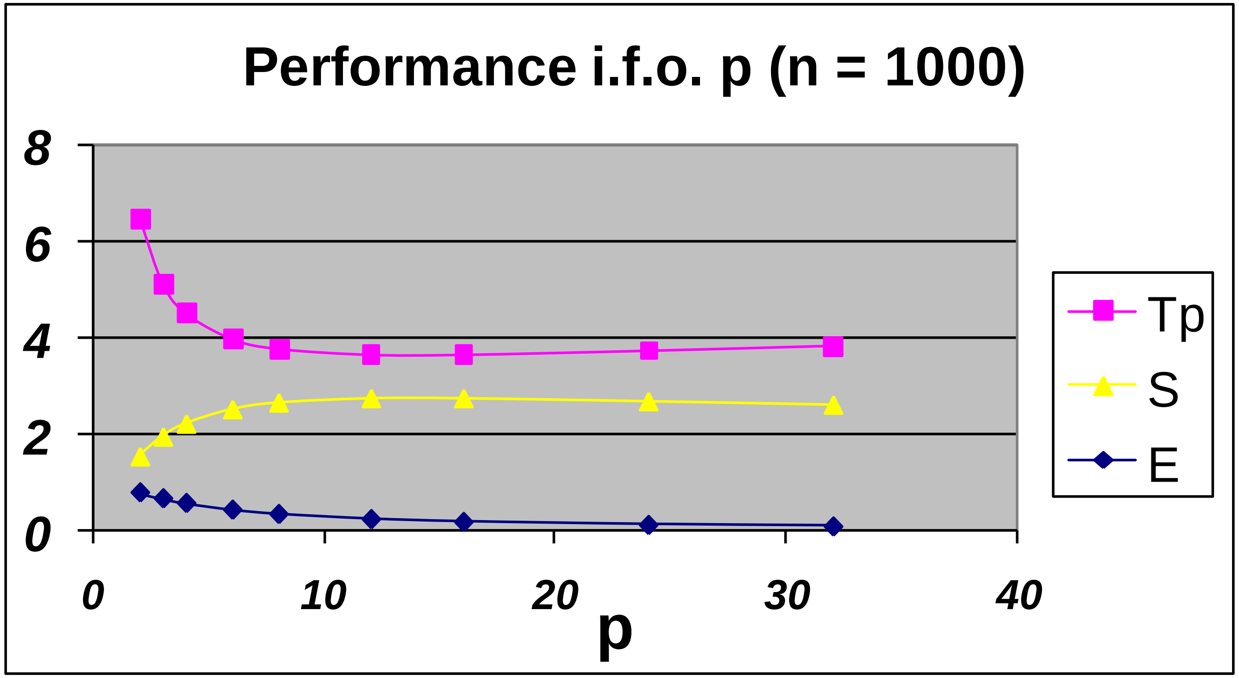
Example 2: Parallel Quicksort



Execution Profile of Parallel Quicksort



Quicksort's performance



Without considering load imbalances

Speedup growth is limited!

Reason?

Amdahl's Law

PPP p. 80

- Limitations of inherent parallelism: a part *s* of the algorithm is not parallelizable

$$T_{seq} = \underbrace{(1-s).T_{seq}}_{\text{parallelizable}} + \underbrace{s.T_{seq}}_{\text{not parallelizable}}$$

$$T_{par} = \frac{(1-s).T_{seq}}{p} + s.T_{seq}$$

$$Speedup_{\max} = \frac{T_{seq}}{T_{par}} = \frac{T_{seq}}{\frac{(1-s).T_{seq}}{p} + s.T_{seq}} = \frac{p}{1+(p-1).s}$$

Assume no other overhead

Amdahl's Law

$$Speedup < \frac{p}{1+(p-1).s}$$

$$Efficiency < \frac{1}{1+(p-1).s}$$

If *p* is big enough:

$$Speedup < \frac{1}{s}$$

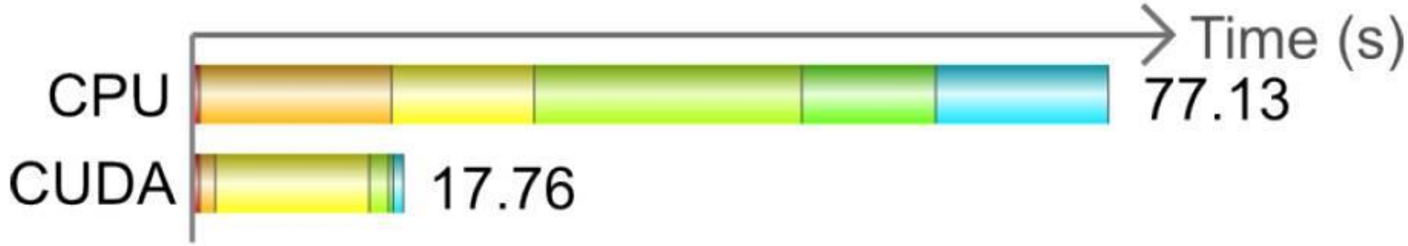
s	Speedup$_{max}$
10%	10
25%	4
50%	2
75%	1.33

Amdahl example: video decoding

Thanks to Wladimir van der Laan, University of Groningen

Decoding 1080p video sequence

Stage	CPU (s)	CUDA (s)	
1 MOTION_DECODE	0.64	0.64	
2 MOTION_RENDER	16.16	1.33	← 12 ×
3 RESIDUAL_DECODE	12.00	12.94	
4 WAVELET_TRANSFORM	22.52	1.63	← 14 ×
5 COMBINE	11.27	0.39	← 29 ×
6 UPSAMPLE	14.53	0.85	← 17 ×
Total	77.13	17.76	← 4.3 ×

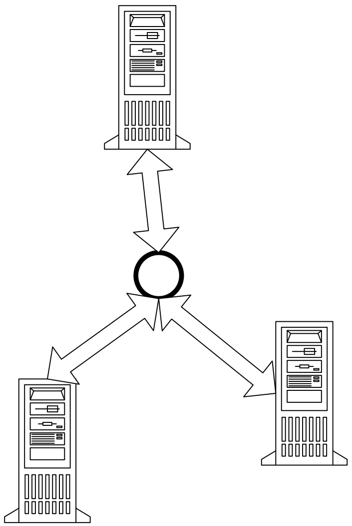


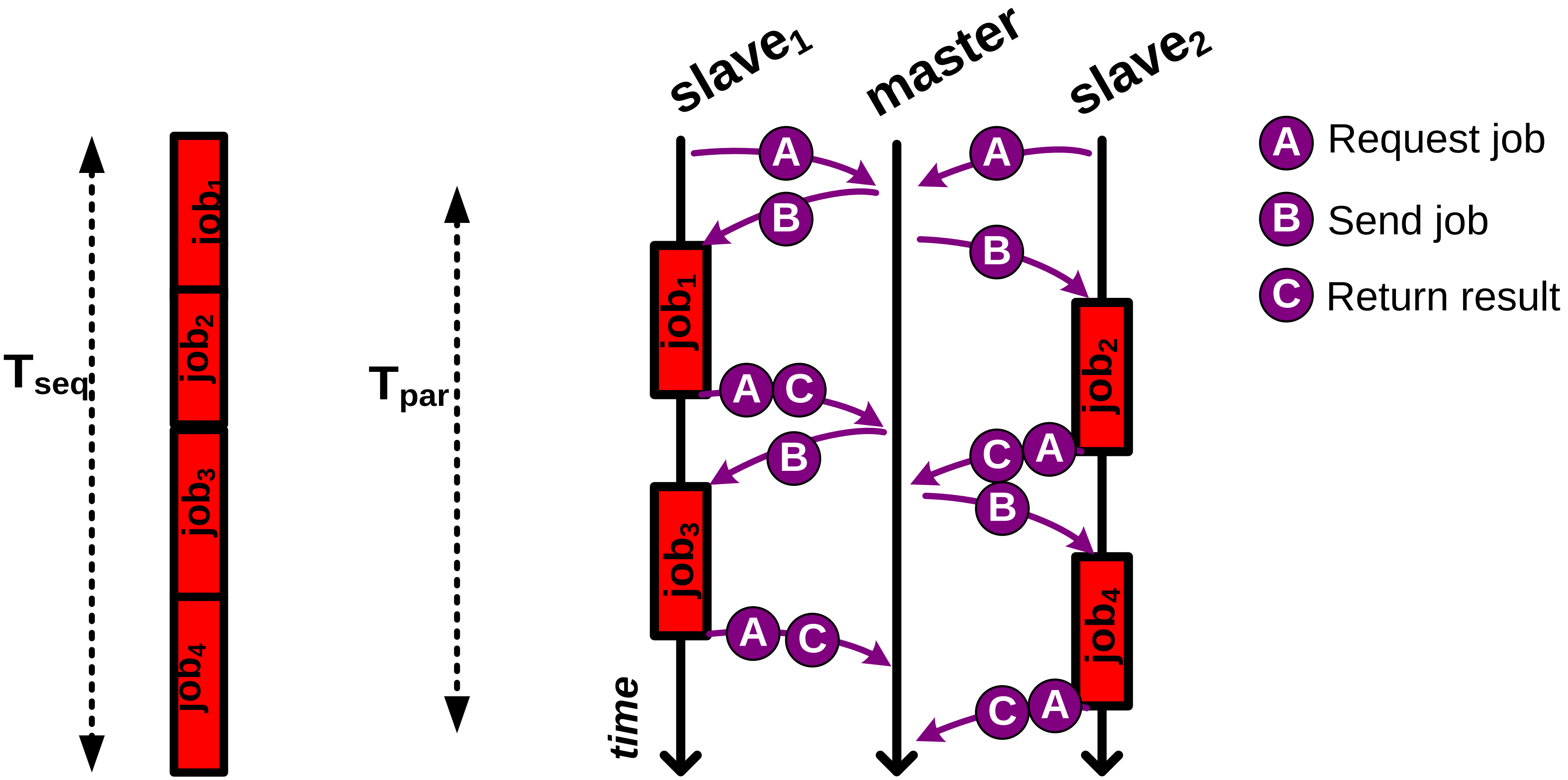
Example 3: Job Farming

Set of jobs & cluster of computers

= *Independent task parallelism*

{job1, job2, job3, job4}




slave1
master
slave2
Request job
Send job
Return result
Tseq
Tpar
job1
job2
job3
job4
time

Speedup = ±1.2

Performance of Job Farming?

Overheads? Bottlenecks?

1. Communication overhead
   - *Impact on speedup ~ T_{seq}/T_{comm} ~ granularity*
   - *Granularity* = computation/communication
   - ⇨ overlap communication with computation
2. Bottleneck at master => idling of slaves
   - ⇨ use several masters (‘tree’-structure)

Scalability

Can we keep efficiency constant while simultaneously increasing W and p?

Table 5.1 Efficiency as a function of n and p for adding n numbers on p processing elements.

n	$p = 1$	$p = 4$	$p = 8$	$p = 16$	$p = 32$
64	1.0	***0.80***	0.57	0.33	0.17
192	1.0	0.92	***0.80***	0.60	0.38
320	1.0	0.95	0.87	0.71	0.50
512	1.0	0.97	0.91	***0.80***	0.62

Scalability

KUMAR p. 211

PPP p. 95

Gustafson's law

- Runtime remains constant if efficiency remains constant and increasing *p* and *W* at the same rate:

$$T_{par} = \frac{T_{seq}}{speedup} = \frac{\alpha . W}{efficiency(W,p).p}$$
$$= \frac{\alpha}{efficiency(W,p)} . \frac{W}{p} = \text{constant}$$

- *Problem doubles? Double processing power! Same time!*
- Program is ***scalable***: the ability to maintain efficiency at a fixed value by simultaneously increasing the number of processors and the size of the problem.
- It reflects a parallel system's ability to utilize increasing processing resources effectively.

Iso-efficiency

$$Efficiency = \frac{1}{1 + \frac{T_{ovh}}{T_{seq}}}$$

iso-efficiency curve:
When is efficiency constant

$$\Rightarrow \frac{T_{ovh}(W, p)}{T_{seq}} = \text{constant} = \frac{T_{ovh}(W, p)}{\alpha . W}$$

If sequential runtime~W

Function tells us **how *W* must increase with an increasing *p* for maintaining efficiency**

- *If perfectly scalable* (T_{ovh} linear or sub-linear in *p*):
  - Increase *W* linearly with increasing *p*
  - Parallel run time stays the same
  - Workload per processor remains constant (see next slide)
- *If fairly/poorly scalable* (T_{ovh} super-linear in *p*):
  - Problem size should be increased more than *p* to keep the efficiency
  - Bigger work load per processor (see next slide)
    - More memory needed!!

scalability of quicksort

$\frac{n}{p}\log_2(n/p)$

$2\,n/p$

$4\,n/p$

n

I n

I n/p

$\approx 2n$

$2n/p$

$\approx n$

$\approx 2n$

$$\frac{T_{seq}}{p\cdot T_{par}} = \frac{n\log_2 n\;\delta_c}{p\left(\frac{n}{p}\log_2\frac{n}{p} + 2n\right)\delta_c + 2np\,\delta_m}$$

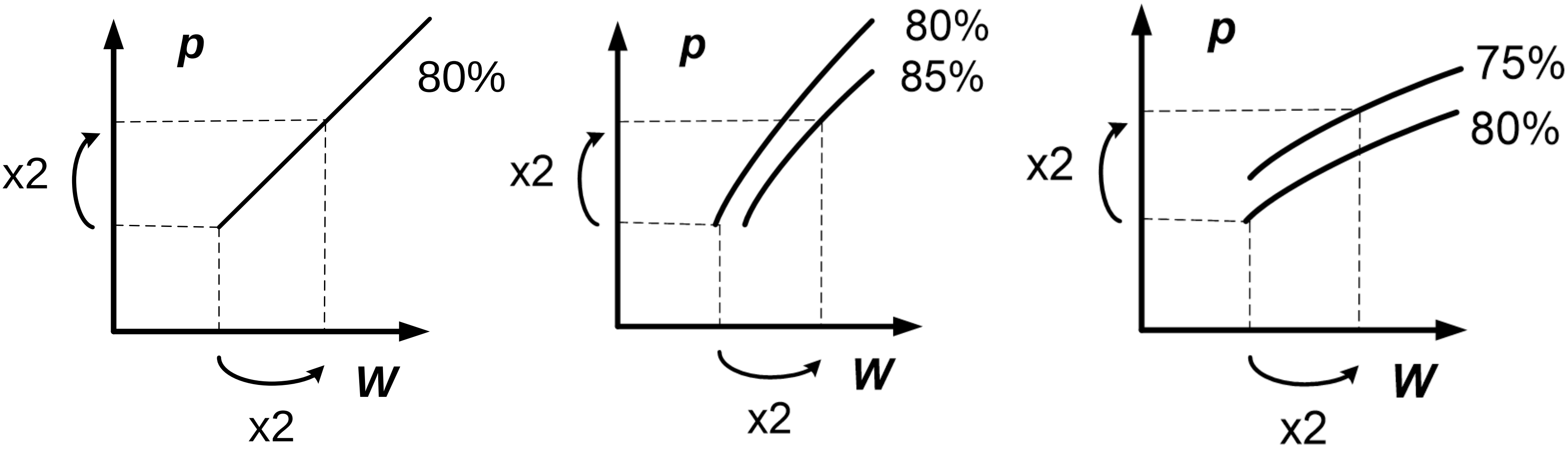
$$\varepsilon = \frac{1}{\frac{\cancel{p}\log_2\frac{n}{p}}{\cancel{p}\log_2 n}\cancel{\delta_c} + \frac{2\cancel{n}p}{\cancel{n}\log_2 n}\cancel{\delta_c} + \frac{2\cancel{n}p}{\cancel{n}\log_2 n}\,\frac{\delta_m}{\delta_c}}$$

$$= \frac{1}{1\,\frac{\log_2 n}{\log_2 n}\cancel{\delta_c} - \underbrace{\frac{\log_2 p}{\log_2 n}\cancel{\delta_c}}_{\text{gets bigger}} + \frac{2p}{\log_2 n}\cancel{\delta_c} + \frac{2p}{\log_2 n}\cdot\frac{\delta_m}{\delta_c}}$$

$$\frac{2p}{\log_2 n}\left(1 + \frac{\delta_m}{\delta_c}\right)$$

$\frac{10}{100}$ $\frac{11}{101}$

Iso-efficiency curves



scalable ***highly scalable*** ***poorly scalable***

Thanks to Noah Van Es (2016)

See Link

Gustafson's law

- **Amdahl's law**: pessimistic view
  - parallelization is limited
  - Amdahl only changes *p*, keeps *W* and serial fraction *s* constant
- **Gustafson**: more optimistic
  - the problems we run in parallel will be bigger and have more parallelism: for higher *p*, higher *W*
    - ➢ *Iso-efficiency curve*
  - Bigger problems: smaller serial fraction, less overhead

Approach to follow

I. Generate/draw execution profile
II. Identify lost cycles
III. Determine causes of overhead
IV. Plot performance in function of p and W
V. Study impact of overheads on speedup
VI. Study scalability
VII. Determine optimization possibilities