Parallel Systems Course: Chapter VII

Parallelization of Discrete Optimization Problems

Advanced Topics in Parallel Processing

Kumar Chapter 11

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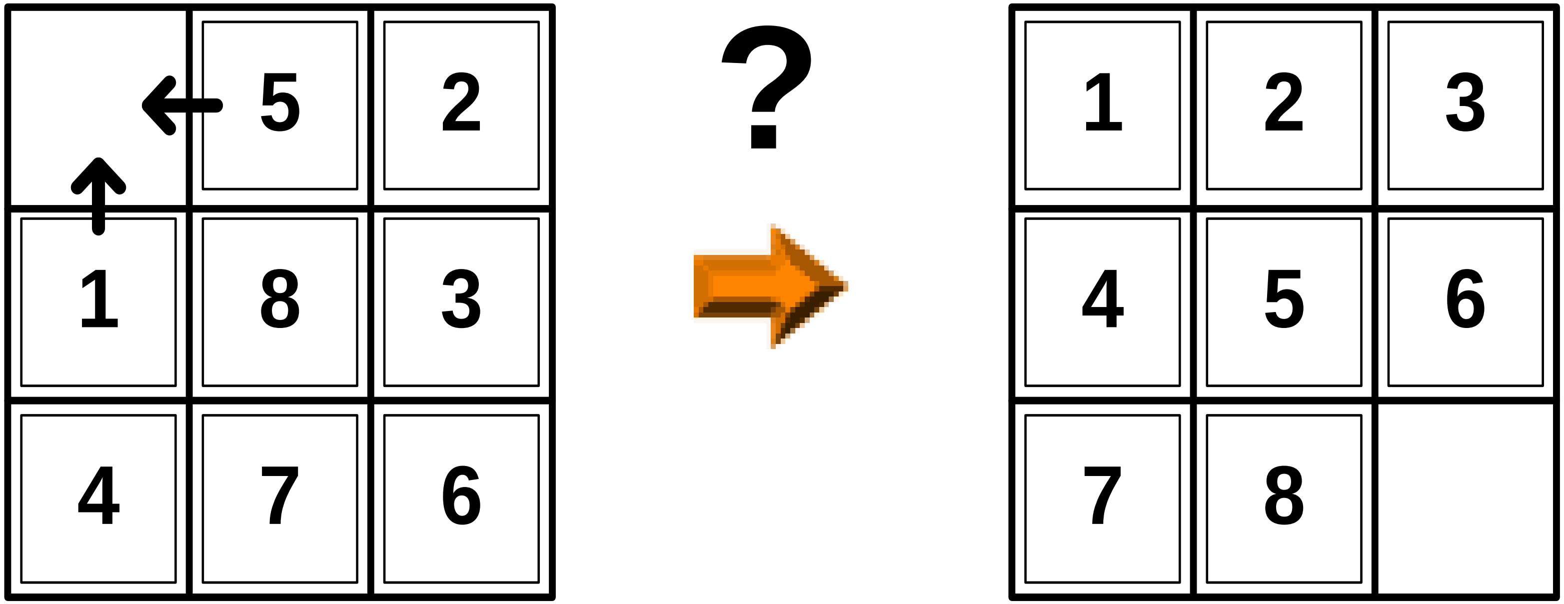
DOP Example: Shift- Puzzle

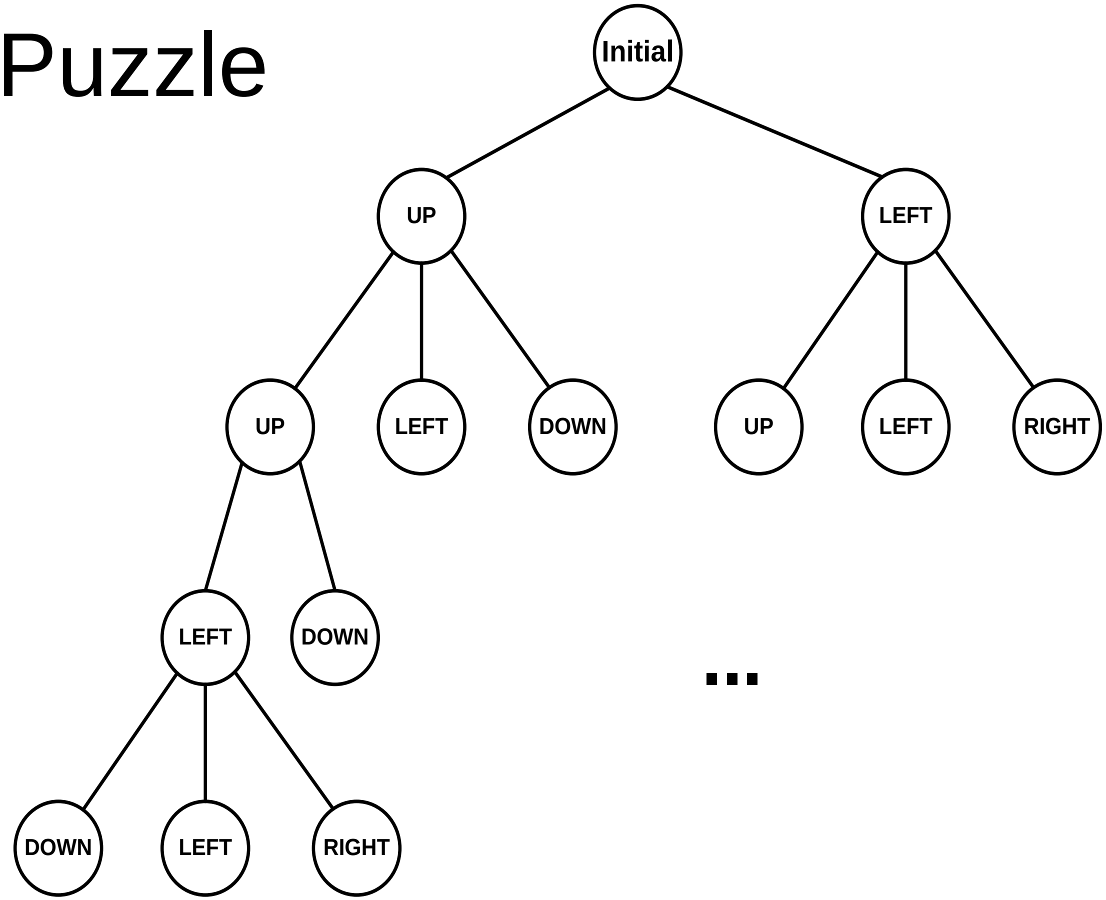


Definition DOP

- Set of feasible solutions *S*
  - Find any feasible solution
- Optional: cost function *f* for each solution
  - Find optimal feasible solution (min or max)
- In terms of complexity of solution methods, there are two classes:
  - Problems that have efficient algorithms for finding optimal solutions.
    ex Dijkstra
  - Problems that don't have such efficient algorithms (NP-complete)
    ex Traveling Salesman Problem
- *Algorithms*
  - Exhaustive search: computational intensive due to large set size
  - Heuristic search

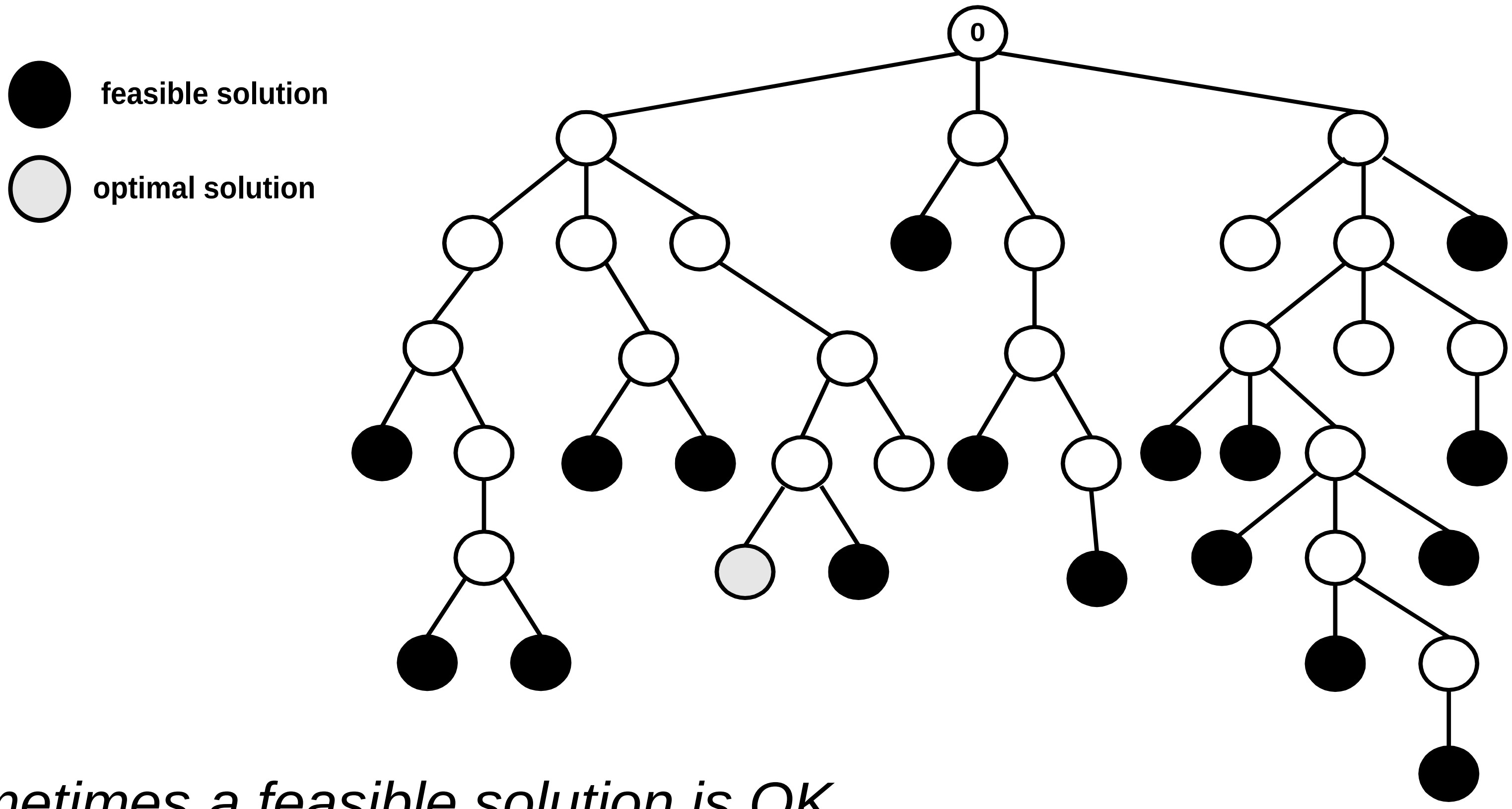
Tree representation

Shift Puzzle



Another example: scheduling of automated warehouses

Tree representation



Sometimes a feasible solution is OK,
in other cases the optimal solution should be found

Sequential Tree Search Algorithms

- Depth – First search
  1. Simple Backtracking
  2. Branch and Bound: limit the depth
- Breadth – First search
  1. Iterative Deepening: with open node list
- Heuristic search
  - **Best first search**:
    - based on breadth-first
    - With heuristic function that identifies promising nodes

Parallelization possibilities

Algorithms

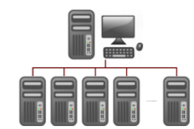
- Brute force
- Branch & Bound
- Iterated Local Search
- …

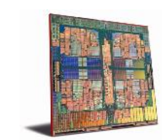
Levels of Parallelism

- Alternative algorithms
- Alternative scenarios (due to uncertainty)
- Multi-search
- Tree-level parallelism
- Node-level parallelism (neighbor exploration)
- parallel calcution of score or bound

Parallel Hardware

- cluster
- multicore
- accelerator





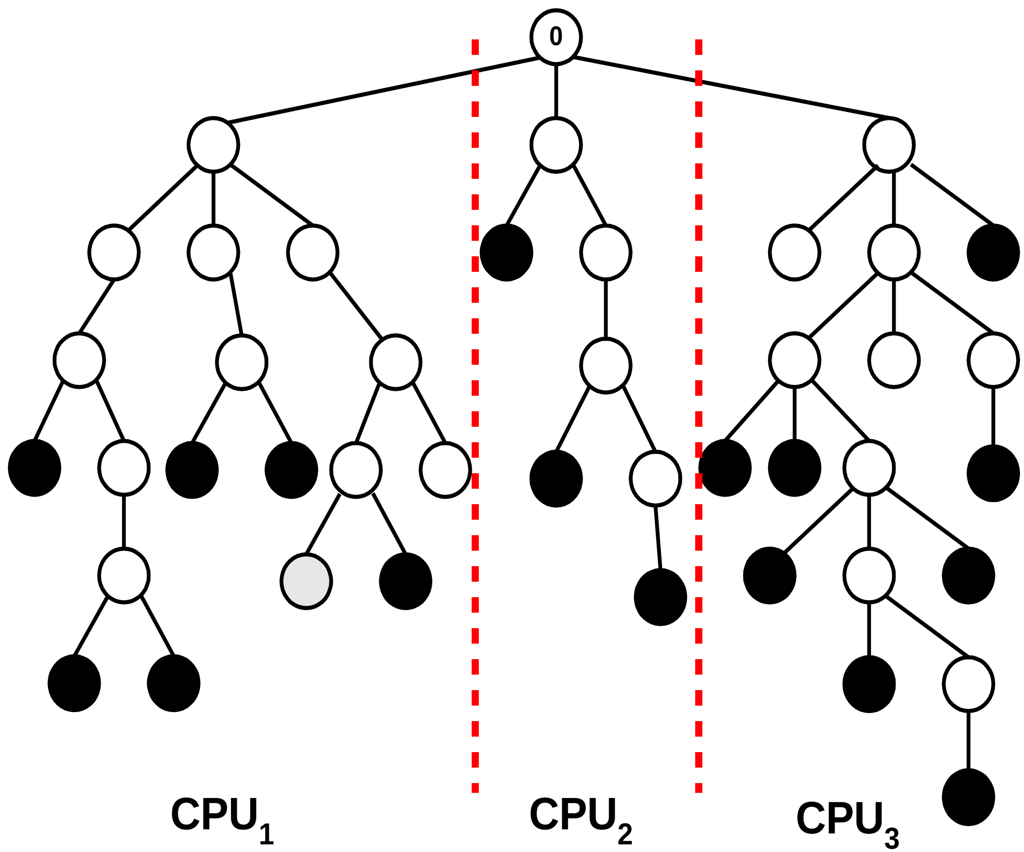
Suitability is problem-dependent!

Our focus here

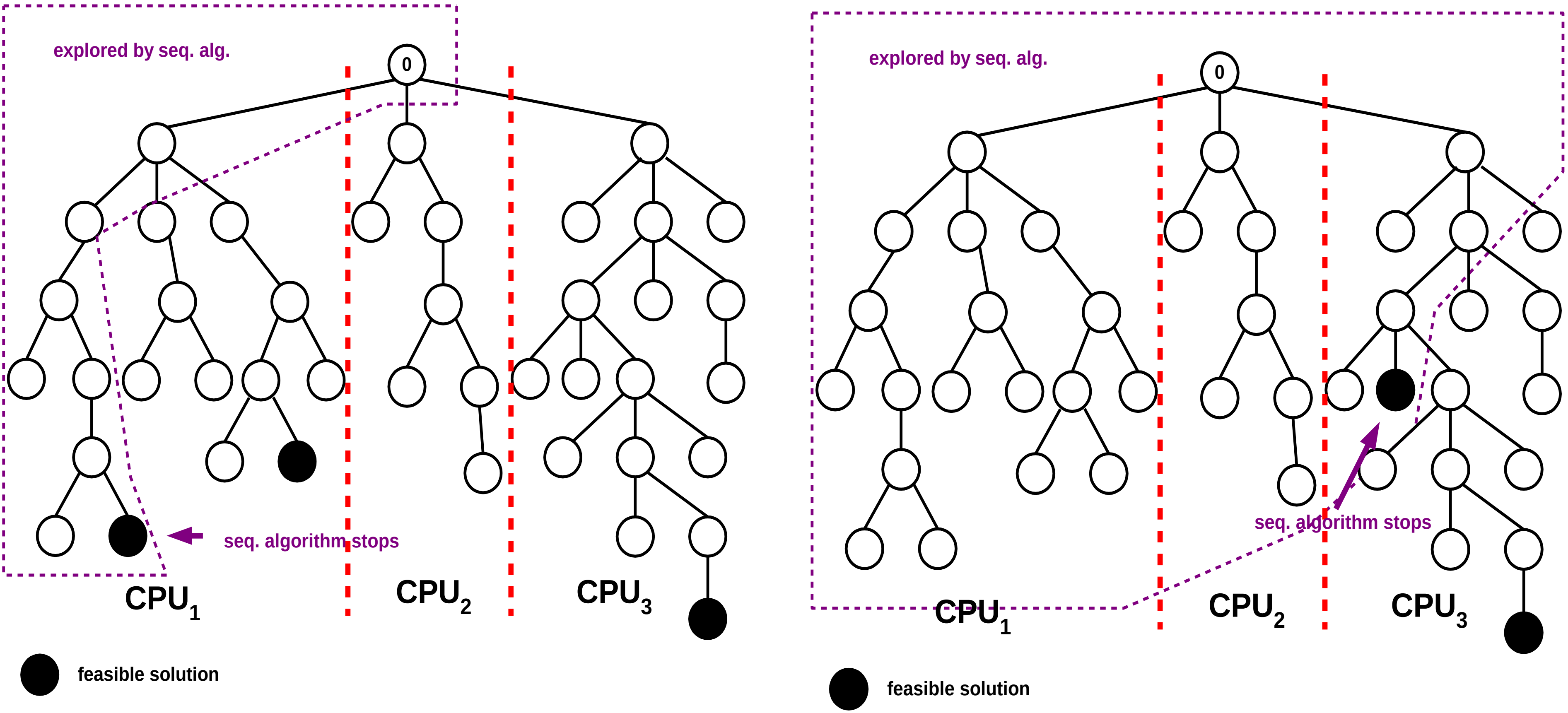
Parallel Depth-first Tree Search

Distribution of tree:

After expansion of root node: send children & problem to slaves



DOP looking for a feasible solution



Parallel Work Anomalies

Sequential work ≠ Parallel work!

$$T_{seq} \neq \sum_{i=1}^{p} T_{work}^{i} \qquad T_{seq} + T_{anomaly} = \sum_{i}^{p} T_{work}^{i}$$

$$Search\ Overhead\ \ Factor = \frac{parallel\ work}{sequential\ work} = \frac{\sum_{i=1}^{p} T_{work}^{i}}{T_{seq}}$$

In our approach: considered as overhead (can be positive or negative)

Impact on overhead: $\dfrac{T_{anomaly}}{T_{seq}}$

Parallel Overhead

- Partitioning: low
- Communication: low
- Synchronization: no
- Returning results: low
- **Idling: HIGH**
  - Due to load imbalances
  - <u>Solution</u>: *dynamic load balancing* **"when finished, ask for work"**
  - Find solutions for:
  1. Donor Selection
  2. Termination Detection

Load imbalances

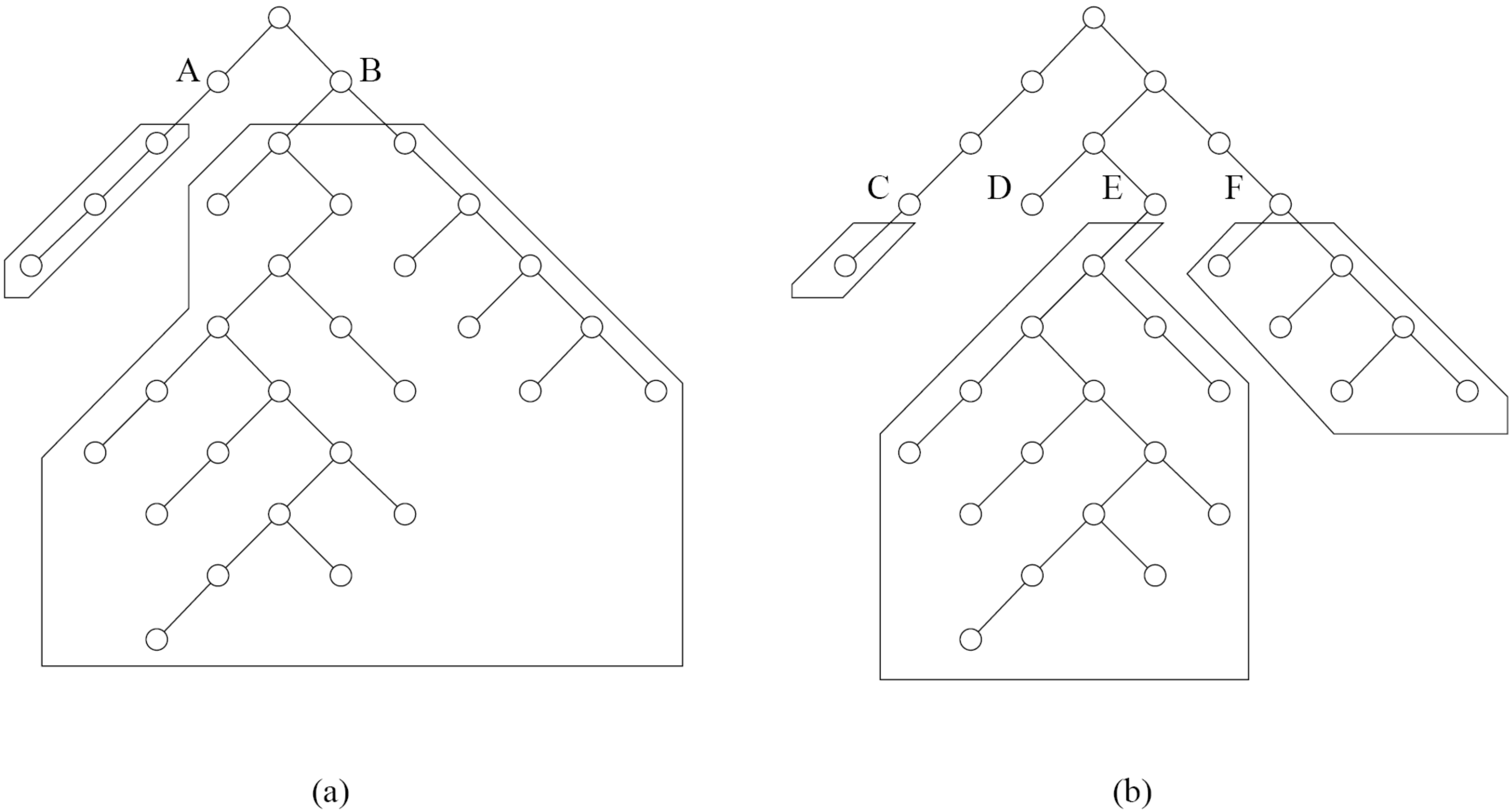


Figure 11.7 The unstructured nature of tree search and the imbalance resulting from static partitioning.

Donor selection

- Asynchronous Round Robin
  Each processor keeps a cyclic list
- Global Round Robin
  Master keeps a cyclic list
- Random polling
  Random selection of donor

Termination detection

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- Via master (centralized)
  - For example: if donor selection happens via master
- Dijkstra's token algorithm (distributed/local)
  - Arrange processes in a ring
  - Without DLB: a simple token that is passed around by processes when they are terminated
  - With DLB:
    - A boolean per process: keeps track whether work has been redistributed since last pass of token
    - A boolean as token: keeps track whether work has been redistributed by one process and the token should go around again
- Tree-Based (partly-distributed)
- other …

Tree-based Termination Detection

- *Idea:* Associates weights with individual work pieces

- **Master starts with weight 1.**
- **It sends work to (p-1) slaves together with weight 1/p for each. It keeps weight 1/p.**
- **If a slave finishes: its sends its weight to master. Master adds it to its weight.**
- **If a slave sends a part of its work, it sends halve of its weight.**
- **Termination when weight at master becomes one and master has finished**

Parallel Best-first Search

Breadth-first: similar to depth-first (every process explores part of the tree)

Best-first: Keep stack of *open nodes*, ordered by a heuristic function

1. **Centralised strategy**: keep stack on master

⇒ send part of nodes to slaves

⇒ Slaves return expanded nodes

A LOT OF COMMUNICATION

2. **Distributed/ local strategy**: stack on each processor

Synchronization of open node list necessary:

- Random communication strategy
- Ring communication strategy
- Blackboard communication strategy

Ring communication

Local

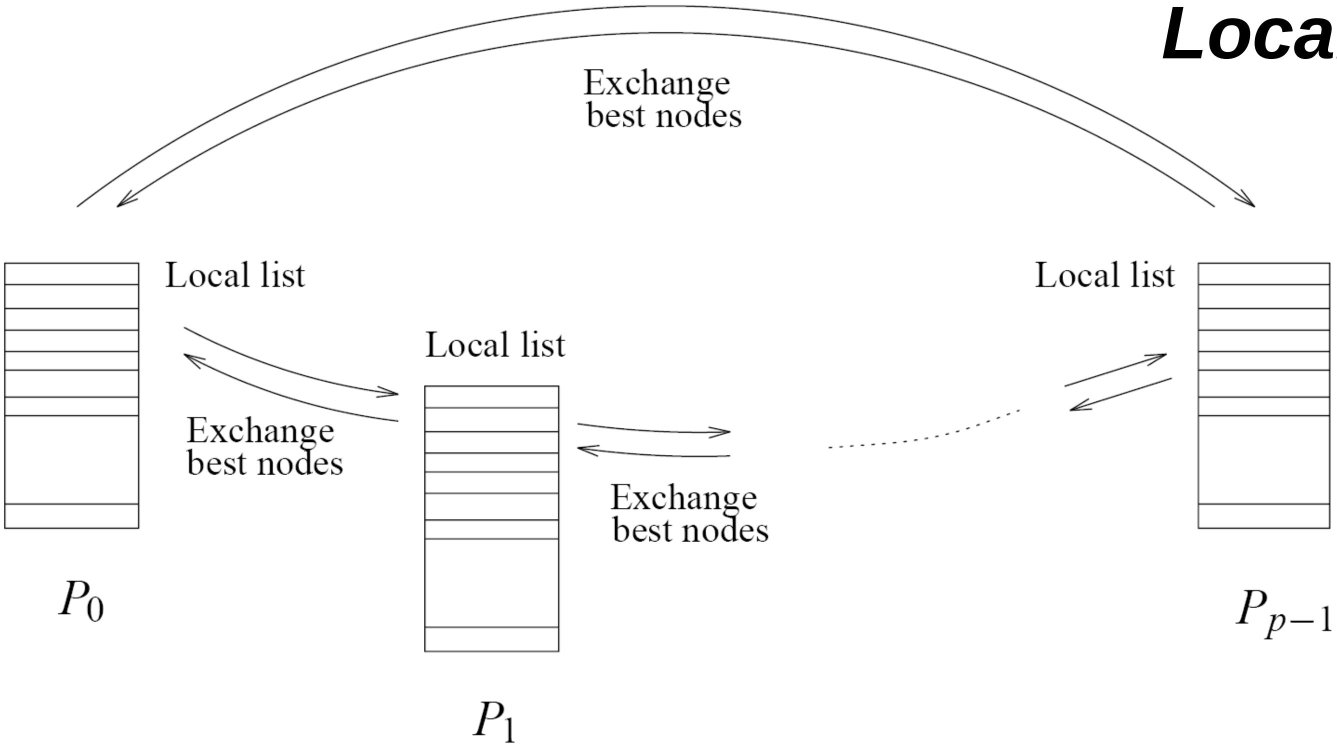

Figure 11.15 A message-passing implementation of parallel best-first search using the ring communication strategy.

Blackboard Strategy

Centralized

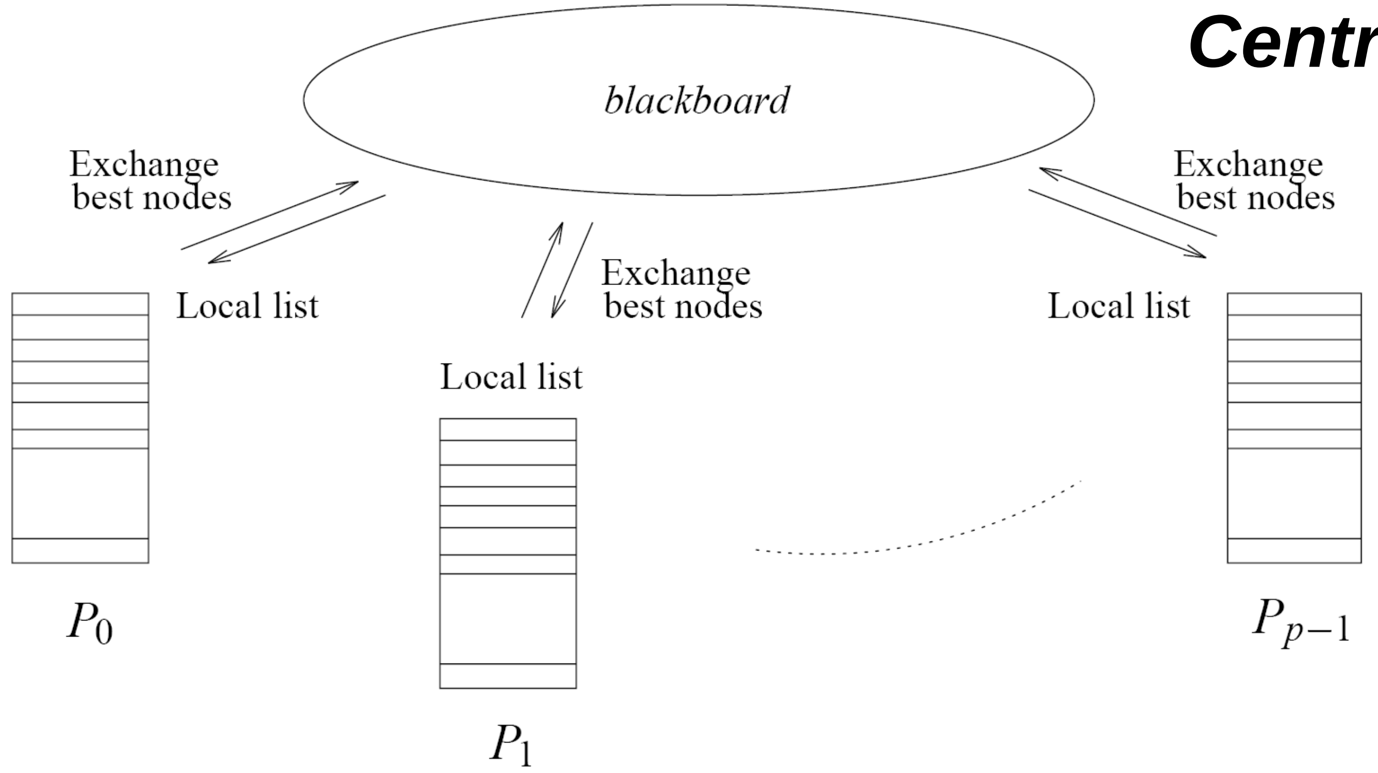

Figure 11.16 An implementation of parallel best-first search using the blackboard communication strategy.

Graph Representation

If the same states can be encountered through different paths (cf puzzle)

Disadvantage of a tree representation: nodes will be checked multiple times!

Solution: Keep a *closed node list*

Check every expanded node whether already visited

- If parallel: synchronization of list (as for open node list)
- retrieve node with a hash function